Assignment 1

ME46085 Mechatronic System Design

by

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Contents

1. Section 1: System Modelling

1.1 Question 1: Equations of Motions

From the free body diagram (Figure 1), the equations of motion can be determined (Equation 1,2,3)

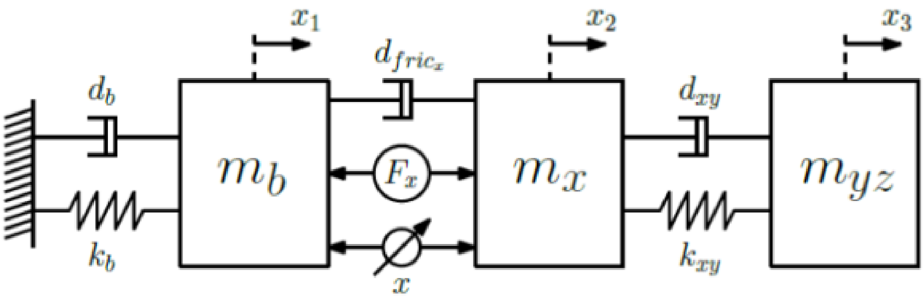

Figure 1: Free Body Diagram

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + k_1x_1 = -f \tag{1}$$

$$m_2\ddot{x}_2 + (c_2 + c_3)\dot{x}_2 - c_2\dot{x}_1 - c_3\dot{x}_3 + k_3(x_2 - x_3) = f \tag{2}$$

$$m_3\ddot{x}_3 + c_3(\dot{x}_3 - \dot{x}_2) + k_3(x_3 - x_2) = 0 \tag{3}$$

1.2 Question 2: Base Mass Analysis

The base mass is required to simulate the rest of the wire-bonding machine apart from the component to be controlled. Since the rest of the machine is not fixed, there is a small but not negligible dynamical contribution from the base mass.

1.3 Question 3 & 4: Transfer Functions and Bode Plot

The equations of motion were rearranged using MATLAB, and then a Laplace transform was performed, assuming all initial conditions $(x_1 = x_2 = x_3 = dx_1 = dx_2 = dx_3) = 0$ (Appendix 1).

Table 1: Mass, damping and stiffness variables

Type	**Base Mass (1)**	**Middle Mass (2)**	**Top Mass (3)**
Mass (m)[kg]	500	2	3
Damping (c)[Ns/m]	6e4	2e2	1e3
Stiffness (k)[N/m]	1e8	0	5e7

The measured values for mass, damping and stiffness were substituted into the transfer function (Equation 4, 5, 6))

$$\frac{\hat{x}_1(s)}{\hat{F}_x(s)} = \frac{-(6s^3 + (5e3)s^2 + (250e6)s)}{(3e3)s^5 + (3.16e6)s^4 + (126e9)s^3 + (20.6e12)s^2 + (25.6e15)s + (1e18)} \tag{4}$$

$$\frac{\hat{x}_2(s)}{\hat{F}_x(s)} = \frac{(3s^2 + (1e3)s + 50e6) * (500s^2 + (60e3)s + 100e6)}{(3e3)s^6 + (3.16e6)s^5 + (126e9)s^4 + (20.6e12)s^3 + (25.6e15)s^2 + (1e18)s} \tag{5}$$

$$\frac{\hat{x}_3(s)}{\hat{F}_x(s)} = \frac{((1e3)s + 50e6) * (500s^2 + (60e3)s + 100e6)}{(3e3)s^6 + (3.16e6)s^5 + (126e9)s^4 + (20.6e12)s^3 + (25.6e15)s^2 + (1e18)s} \tag{6}$$

Substituting $s = -j\omega$, the magnitude $|\frac{\hat{x}_n(s)}{\hat{F}_x(s)}|$ and phase $\angle\frac{\hat{x}_n(s)}{\hat{F}_x(s)}$ of each degree of freedom in a Bode Plot (Figure 8,3,4).

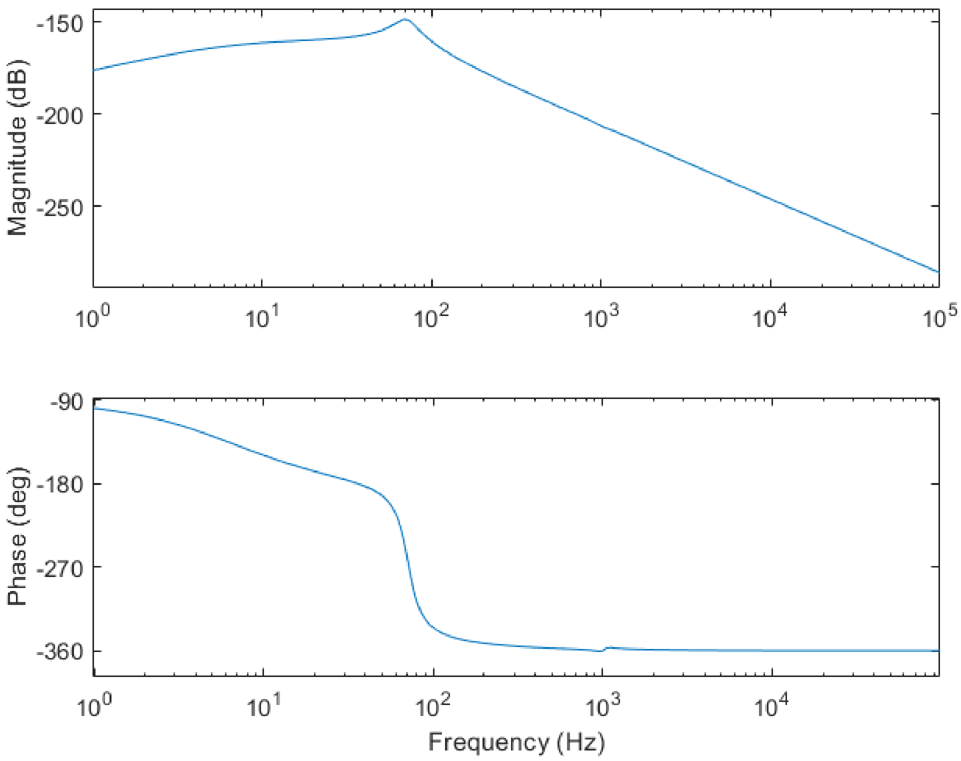

Figure 2: Bode Plot for $\frac{\hat{x}_1(s)}{\hat{F}_x(s)}$

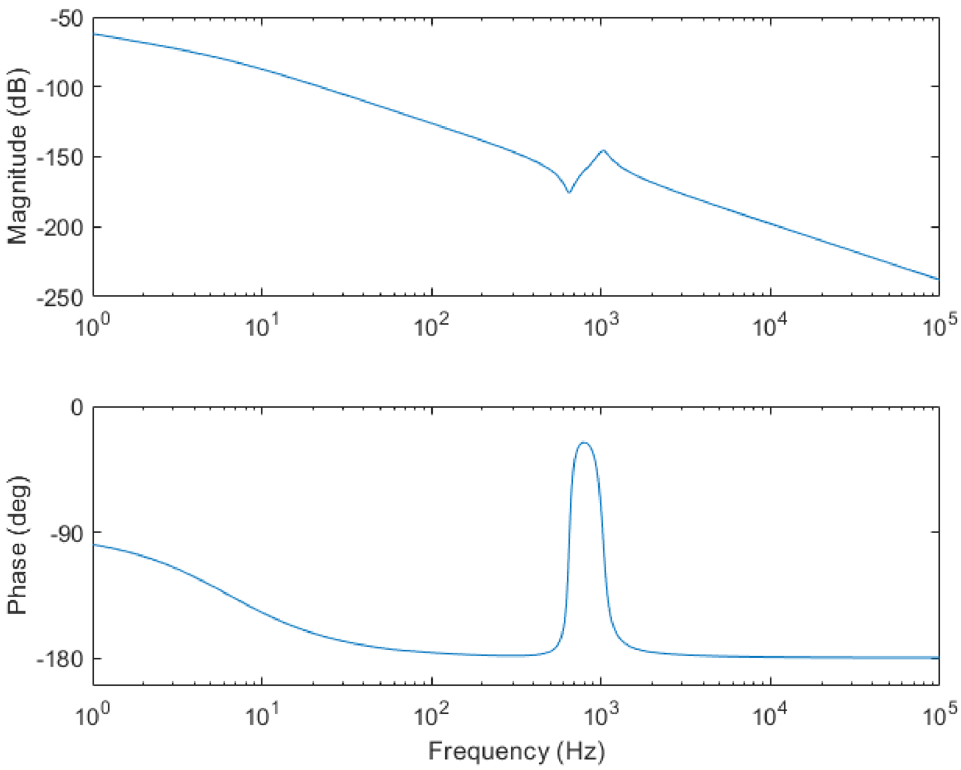

Figure 3: Bode Plot for $\frac{\hat{x}_2(s)}{\hat{F}_x(s)}$

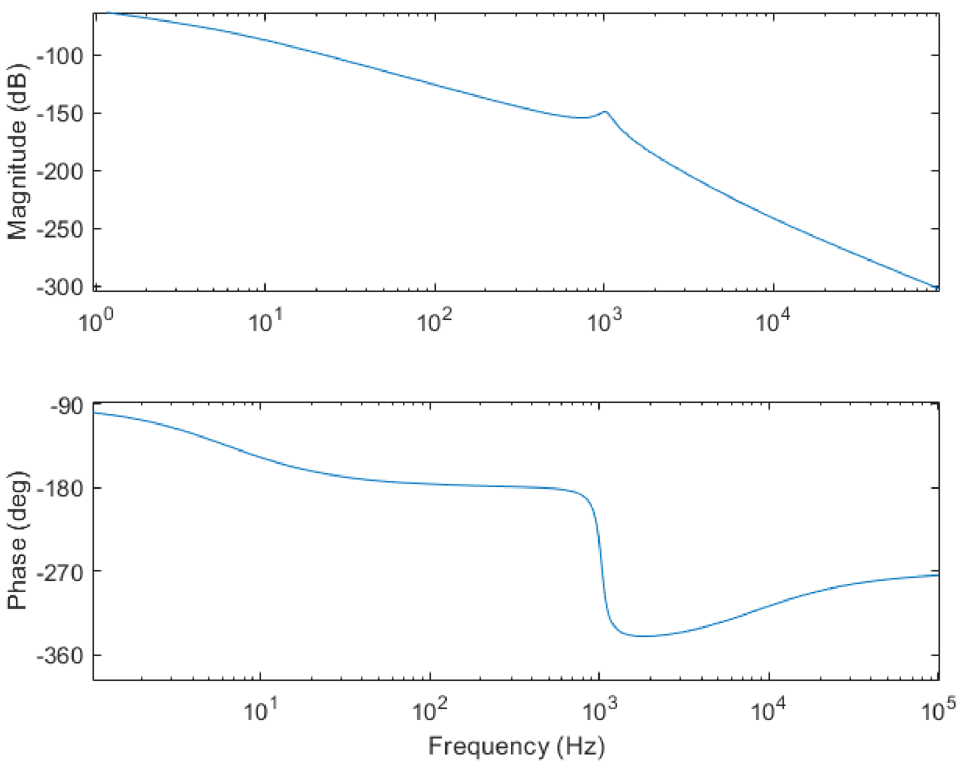

Figure 4: Bode Plot for $\frac{\hat{x}_3(s)}{\hat{F}_x(s)}$

Both the x_2 and x_3 transfer functions have a phase of -90 degrees at low frequency and resonant peak around 1000Hz. However, x_2 also has an anti-resonance and has a phase of -180 degrees at high frequency, compared to the -270 degrees of the x_3 transfer function. Mathermatically, x_2 and x_3 are identical except for a $3s^2$ term in the x_2 numerator.

1.4 Question 5: Modal Analysis

Equations 1,2,3 can be written in matrix form (Equation 7)

$$\begin{bmatrix} -f_x \\ f_x \\ 0 \end{bmatrix} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 & -c_3 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 & 0 & -k_3 \\ 0 & k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{7}$$

The eigenvalues and eigenvectors were then solved for the undamped system (Equation 8).

$$(\mathbf{inv(M)K} - \lambda \mathbf{I})\vec{\nu} = 0 \tag{8}$$

Where **M** and **K** are the mass and stiffness matrices respectively, **I** is the identity matrix. λ are the eigenvalues and $\vec{\nu}$ are the eigenvectors. (Table 2)

Table 2: Natural Frequencies

Eigenvalues	0	200000	41666666
Frequency (rad/s)	0	447.2	6454
Frequency (Hz)	0	71.2	1027.3

The aforementioned transfer functions can be simplified by neglecting damping. For x1 at low frequencies, the transfer function is $\approx \frac{1}{k_1}$, giving a zero slope and $-180°$ phase in Figure 5. The single resonant peak occurs at 71.2 Hz and 70.3Hz for the undamped and damped system respectively. This is then followed by a slope of -2 and a decrease in phase to $-360°$, as the transfer function is $\approx \frac{1}{m_1 s^2}$. It is notable at low frequencies, the damped transfer function has a $-90°$ phase difference and a lower amplitude, so this simplification is only valid for frequencies above 10 Hz.

$$\frac{\hat{x}_1(s)}{\hat{F}_x(s)} = \frac{1}{m_1 s^2 + k_1} \tag{9}$$

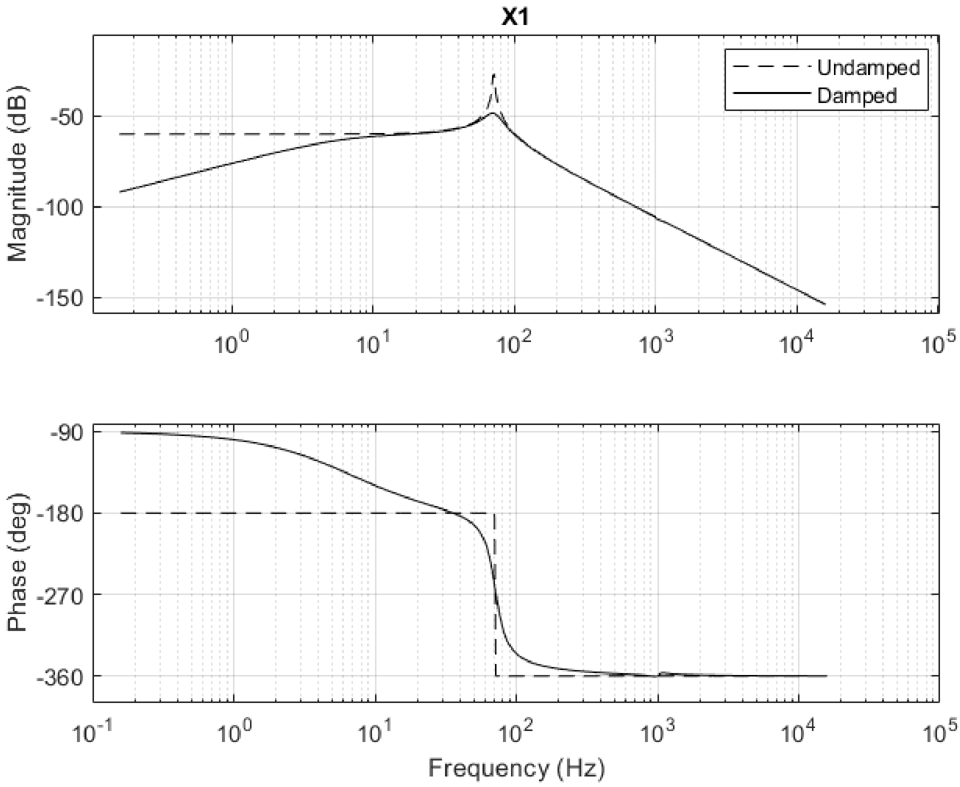

Figure 5: Bode plot comparison of undamped and damped system for x1

For x2 at low frequencies, the transfer function has a slope of -2 and phase of $-180°$, as the transfer function is $\approx \frac{1}{(m_2+m_3)s^2}$. An anti-resonance occurs at 644.1Hz Figure 6 causing the phase to increase to $0°$. This is followed by a resonance peak at 1022Hz and a continuation of the -2 slope since the transfer function is $\approx \frac{1}{m_2 s^2}$. This also coincides with the phase returning to $-180°$. Again, the damping has a significant affect below 10Hz.

$$\frac{\hat{x}_2(s)}{\hat{F}_x(s)} = \frac{(m_3 s^2 + k_3)}{(m_2+m_3)k_3 s^2 + m_2 m_3 s^4} \tag{10}$$

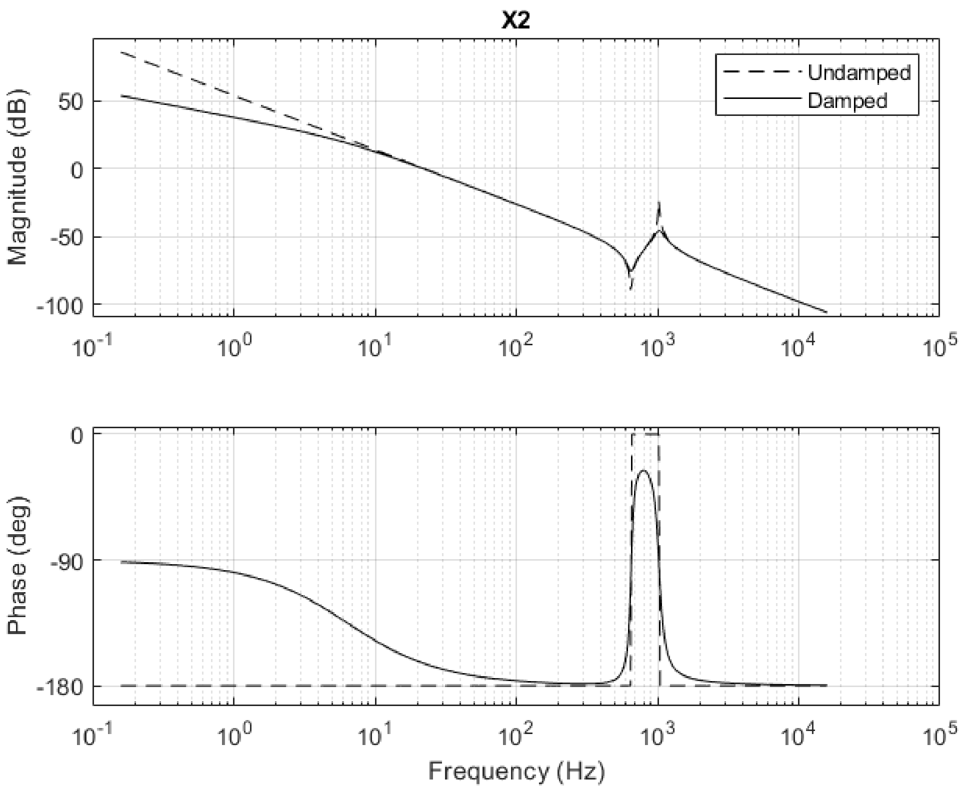

Figure 6: Bode plot comparison of undamped and damped system for x2

At low frequencies, x3 has a slope of -2 since the transfer function is $\approx \frac{1}{(m_2+m_3)s^2}$. A single resonance peak occurs at 1022Hz. At high frequencies, the transfer function is $\approx \frac{k_3}{(m_2 m_3)s^4}$, giving a slope of -4 and a phase of $-360°$. Likewise, below 10Hz the damping starts to dominate.

$$\frac{\hat{x}_3(s)}{\hat{F}_x(s)} = \frac{k_3}{(m_2+m_3)k_3 s^2 + m_2 m_3 s^4} \tag{11}$$

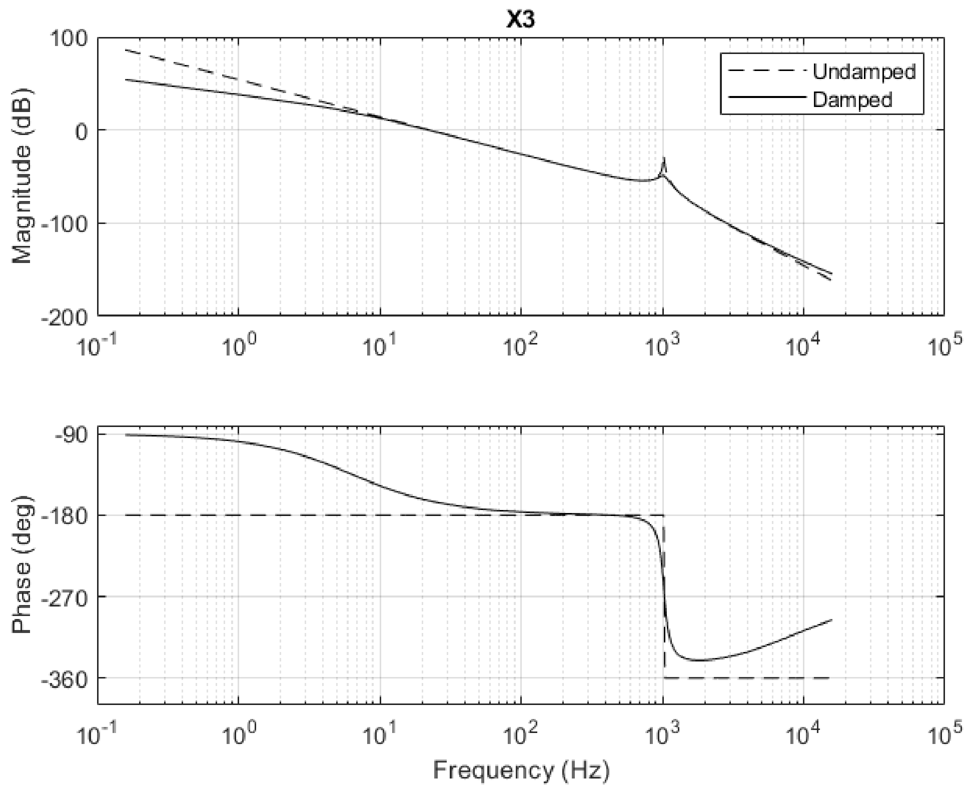

Figure 7: Bode plot comparison of undamped and damped system for x3

2. Section 2: System Identification

2.1 Question 6: Identifying Plant

The input into the unknown plant was a logarithmic chirp. It was chosen to excite a wide range of frequencies but also to be simple to compute. It was also repeated two more times to to capture the response from high to low frequency, not just low to high frequency. A hanning window was used to process the signal.

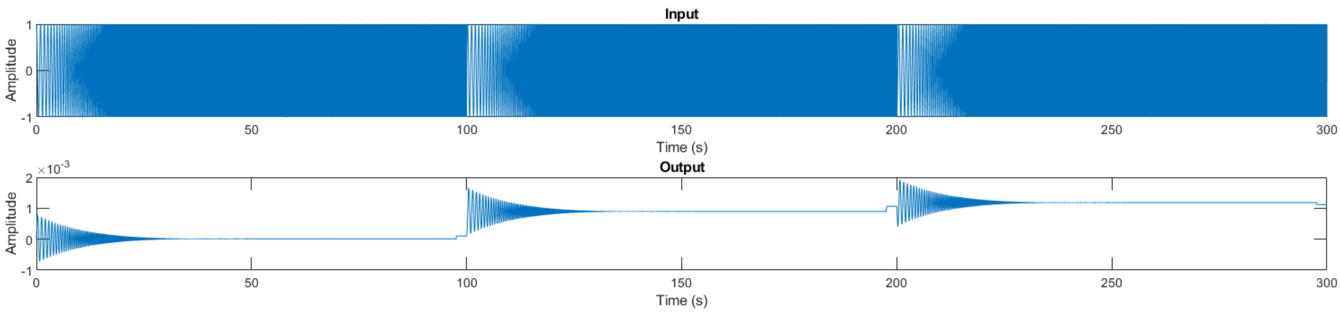

Figure 8: Input and Output in unknown transfer function

Using MATLAB's signal processing toolbox, the unknown transfer function was estimated. The coherence of the estimation was calculated to be 1 for all frequencies (Figure 9), however this is totally unrealistic as beyond 4000Hz there is a considerable amount of noise.

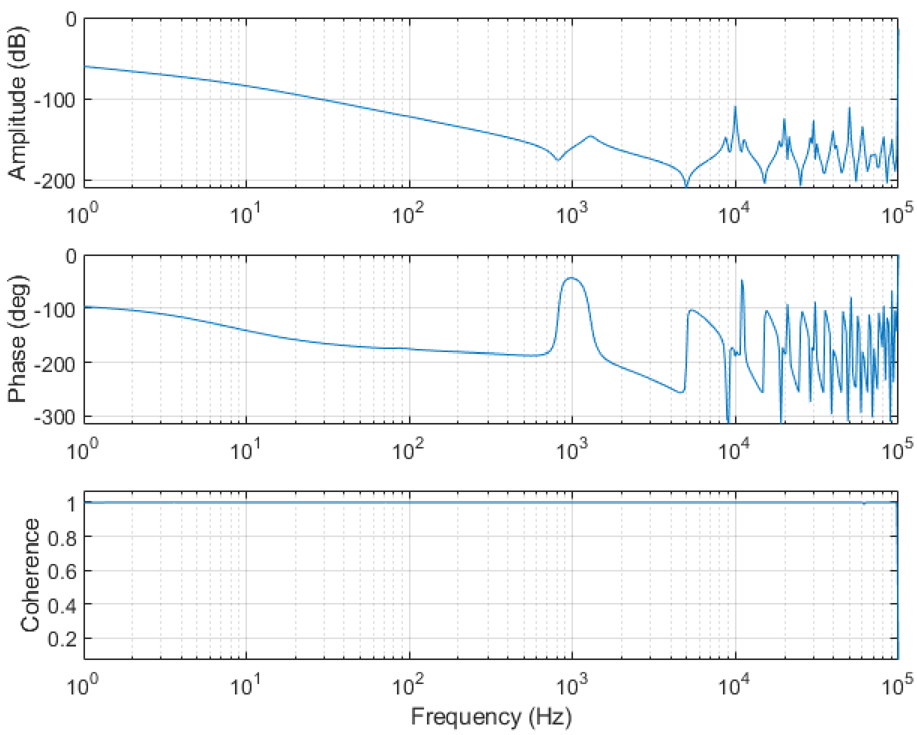

Figure 9: Bode Plot of unknown transfer function

2.2 Question 7: Comparing Transfer Functions

The unknown transfer function is very similar to the x2 (Equation 10) transfer function (Figure 10). However, it is slightly shifted to higher frequencies, with an anti-resonance around 800Hz and a resonance at 1300 Hz.

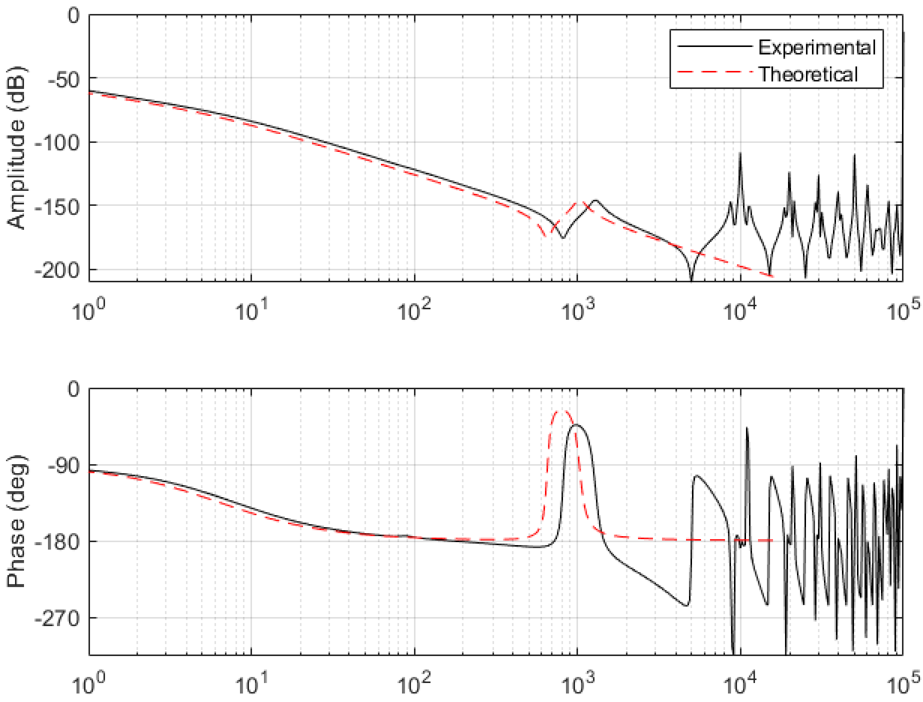

Figure 10: Experimental (unknown TF) vs Theoretical (from 1st principles)

2.3 Question 8: Transfer Function with Delay

The theoretical transfer function is subjected to a delay of t = 0.125ms (Equation 13).

$$G_t = e^{-ts} \tag{12}$$

This was combined in series with the theoretical transfer function (Equation 5) for the plant.

$$G_d = G_t G_p = e^{-ts}\frac{\hat{x}_2}{F_x} \tag{13}$$

This has no effect on the amplitude of the system (Figure 11), however it reduces the phase below $-180°$ instead of forming an asymptote. Noise dominates at frequencies over 400Hz.

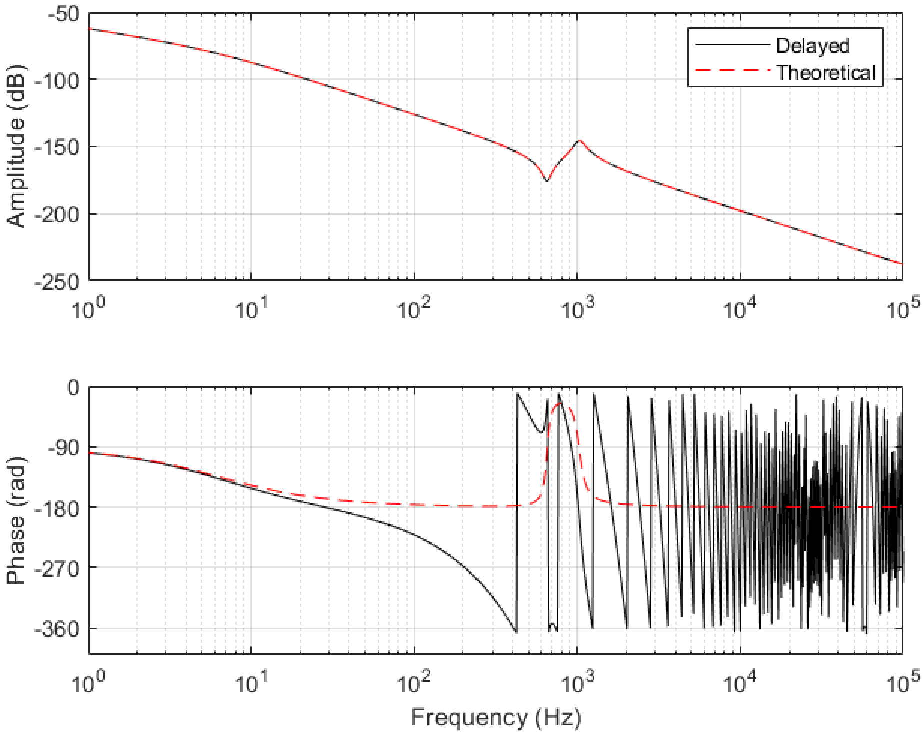

Figure 11: Bode plot of Delayed vs Theoretical Transfer Function

3. Section 3: Controller Design

3.1 Question 9: PID Controller Design

The controller was designed to be robust to prevent instabilities occurring. This involved ensuring a phase margin of >= 30°, a gain margin >= 6 dB and a modulus margin <= 6 dB.
The PID controller has a transfer function:

$$G_c = k_p + \frac{k_i}{s} + \frac{Nk_d}{1 + \frac{N}{s}} \tag{14}$$

Where k is the gain, for the proportional (p), integrator (i) and differentiator (d). The derivative at high frequencies is tamed by a low pass filter, defined by the filter coefficient N, to reject noise. No other filters were required as the resonant peak of the controller is higher than the bandwidth (Figure 14)

Initially, a controller was designed using the rule of thumb (Equation 15, 16). This was then optimised so the controller would meet the required safety margins (Figure 12,13).

$$G_c = k_p(1 + \frac{\omega_i}{s})\frac{(\frac{s}{\omega_d} + 1)}{(\frac{s}{\omega_t} + 1)}; \tag{15}$$

$$\omega_i = \frac{\omega_a}{10}, \quad \omega_d = \frac{\omega_a}{a}, \quad \omega_t = a\omega_a, \quad k_p = \frac{1}{a}\frac{1}{|G_d(j\omega)|} \tag{16}$$

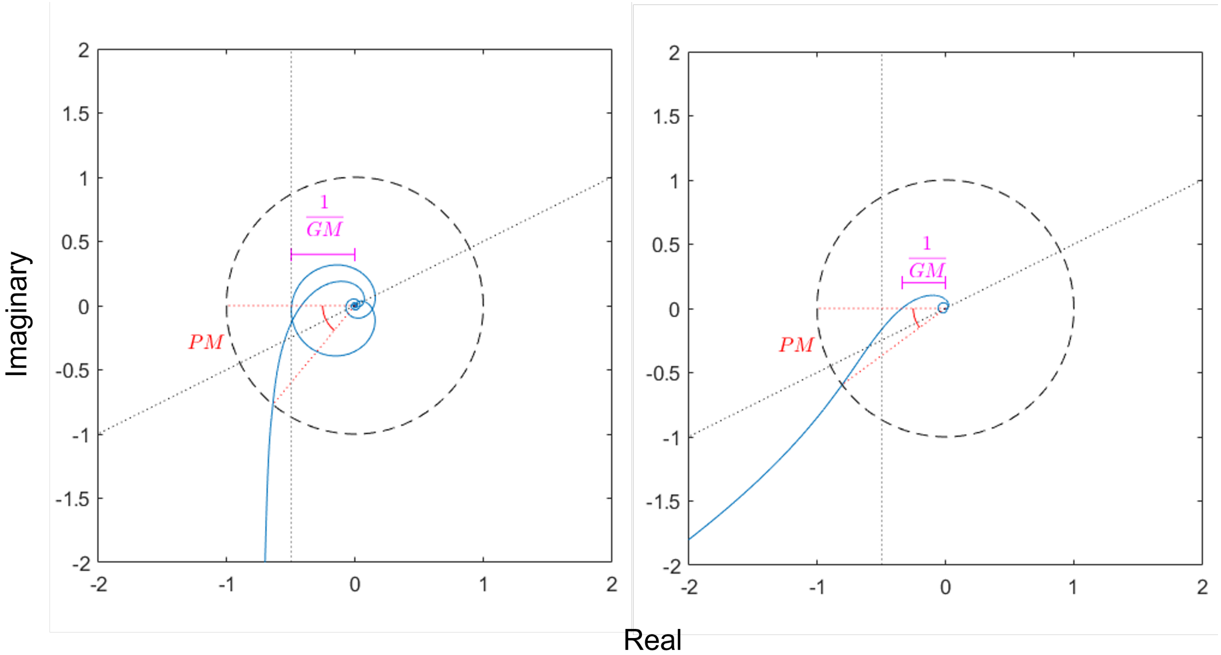

Figure 12: Optimised (L) vs Rule of Thumb (R) Nyquist

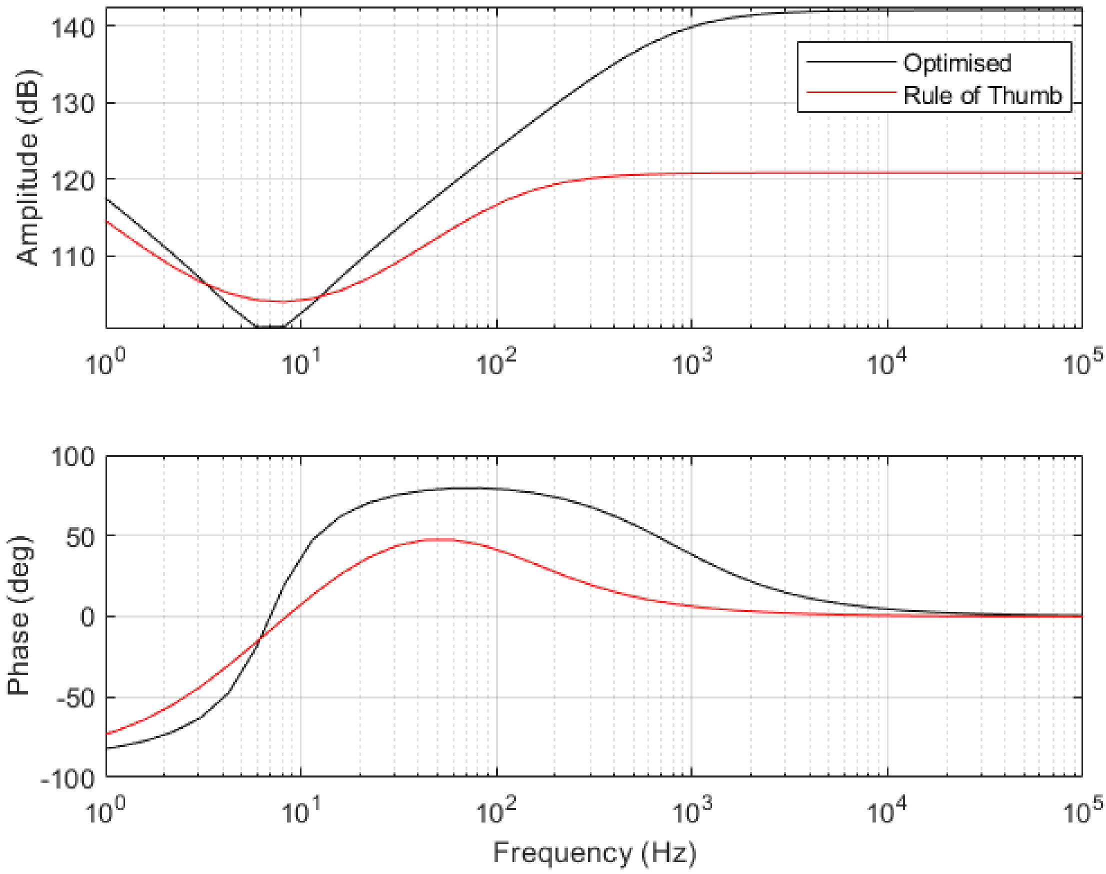

Figure 13: Optimised vs Rule of Thumb Bode

3.2 Question 10: Stability of Controller

The phase margin (PM) was defined to be the angle between the horizontal and the unit circle intersection on the nyquist plot (Figure 14). The gain margin (GM) was defined to be the reciprocal of the real part of the intersection with the horizontal. The modulus margin (MM)was defined to be the maximum value of the sensitivity function. The bandwidth was defined to be the frequency at which the combined controller and delayed plant amplitude = 0 dB. Note: the dotted black lines represent the limits of the margins specified earlier.

The corresponding values are shown in (Table 3).

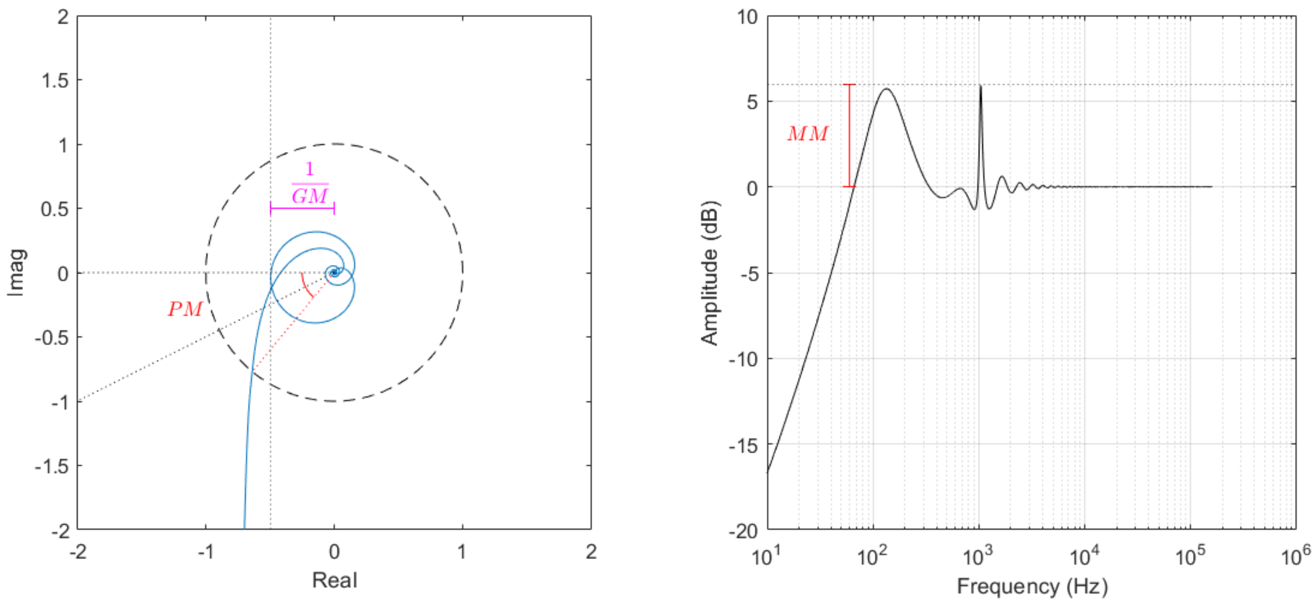

Figure 14: Phase, Gain and Modulus Margin

Table 3: Controller Frequency Domain Parameters

Proportional Gain [k_p]	1e+05
Integrator Gain [ki]	1.5e+06
Differentiator Gain (Hz) [k_d]	2.4e+03
Filter Coefficient [N]	5e+03
Phase Margin ($^\circ$)	50.4
Gain Margin(dB)	6.17
Modulus Margin (dB)	5.91
Bandwidth (Hz)	74.9

The open loop transfer function is the combination of the delayed and controller transfer function in series (Equation 17).

$$G_L = G_d G_c \tag{17}$$

This is shown in (Figure 15).

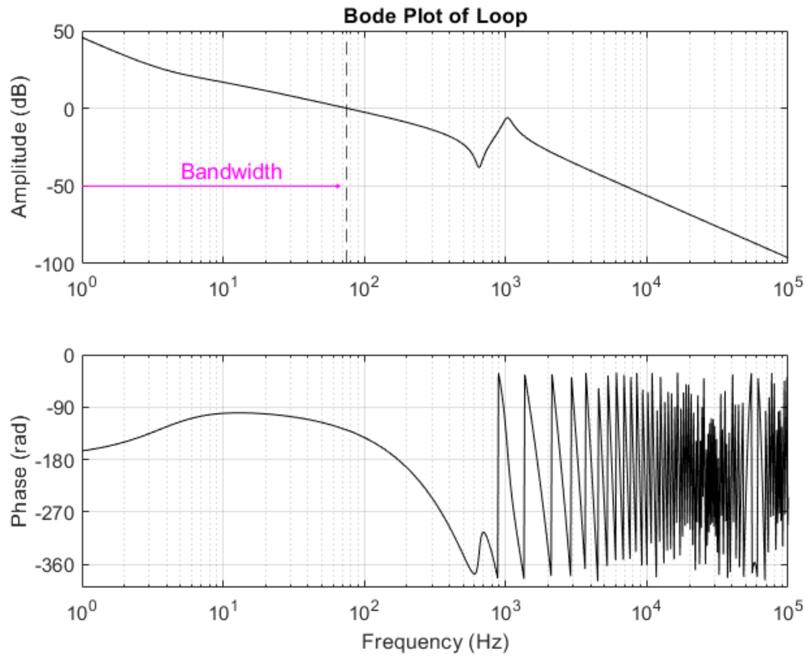

Figure 15: Bode Plot of the Loop

The sensitivity (S) and complimentary sensitivity (T) (Equation 18 is shown in Figure 16. The stability is shown in Figure 14.

$$S = \frac{1}{1 + G_L}, \quad T = \frac{G_L}{1 + G_L} \tag{18}$$

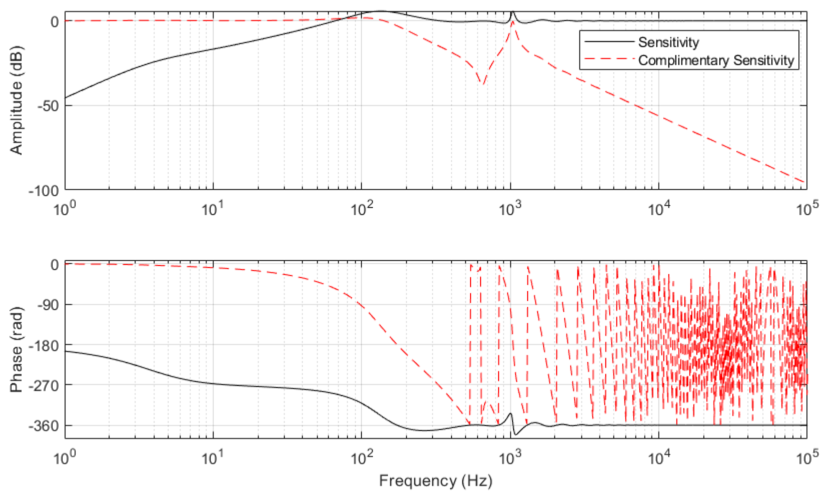

Figure 16: Bode Plot of the Sensitivity

3.3 Question 11: Following Reference Signal

The controller was discretised for a sample time $t_s = 0.125$ms (Equation 19).

$$G_{c,d} = k_p + \frac{k_i t_s}{z-1} + \frac{N k_d}{1 + \frac{N t_s}{z-1}} \tag{19}$$

It was then implemented in a feedback loop (Figure 17) in Simulink.

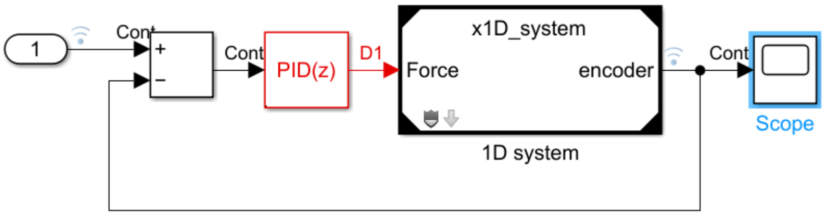

Figure 17: Simulink Diagram Feedback

When following the reference signal it had a maximum overshoot of 1.13% at 0.15 seconds and a maximum undershoot of 0.01% at 0.3 seconds (Figure 18).

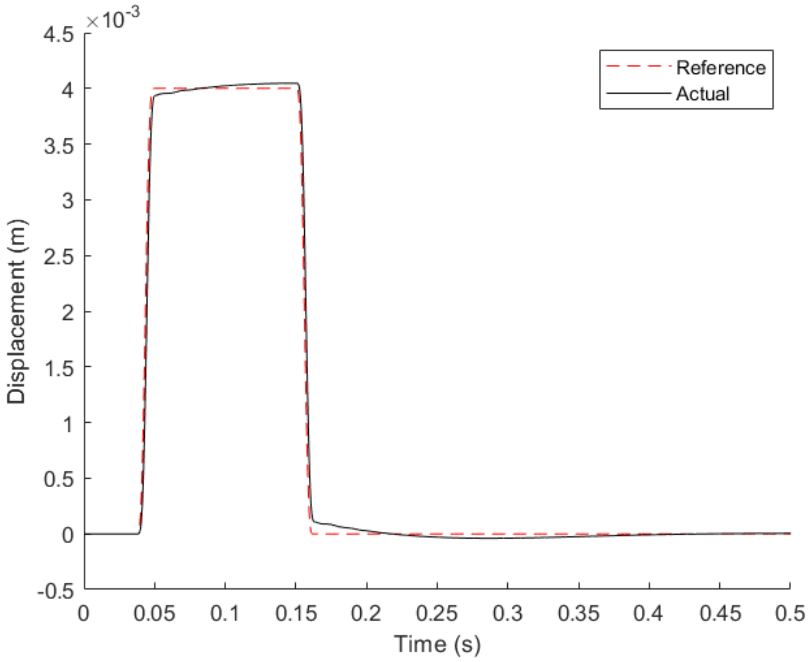

Figure 18: Simulink Diagram Feedback

3.4 Question 12: Step Response

For a step response (Figure 19) for a time sampling of 0.125ms, the overshoot is 2.87% occurring 3.25ms after the perturbation. The settling time, defined as the time to reach 98% of the final value is 8.25 ms.

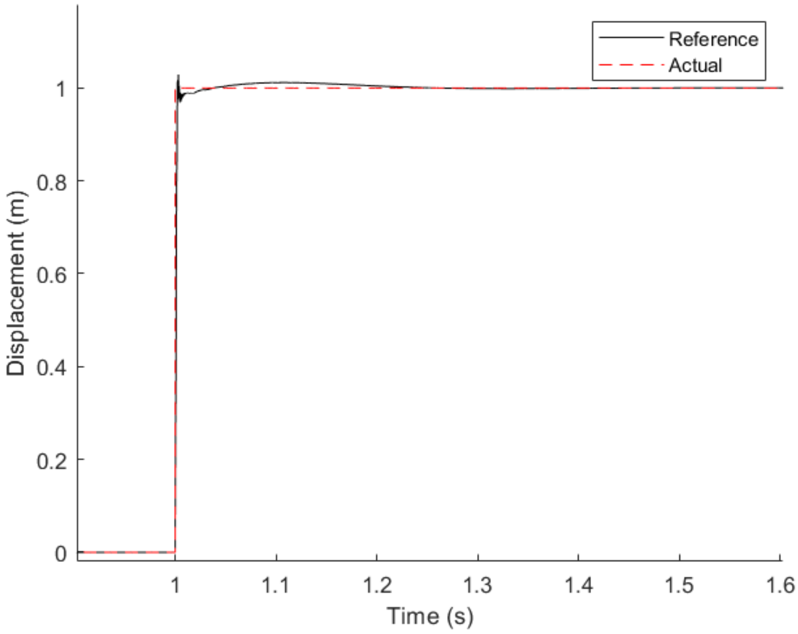

Figure 19: Step Response of hidden system

3.5 Question 13: Discrete Sensitivity

Comparing the two graphs in Figure 20, the discrete sensitivity stops at frequencies above around 3000Hz. Before the resonant peak, the amplitudes are identical, however afterwards the amplitude of the sensitivity of the continuous system increases whereas in the discrete system it plateaus. The opposite occurs in the phase as the discrete system increases to 180° at 3000Hz whereas the continuous system tends towards 90°.

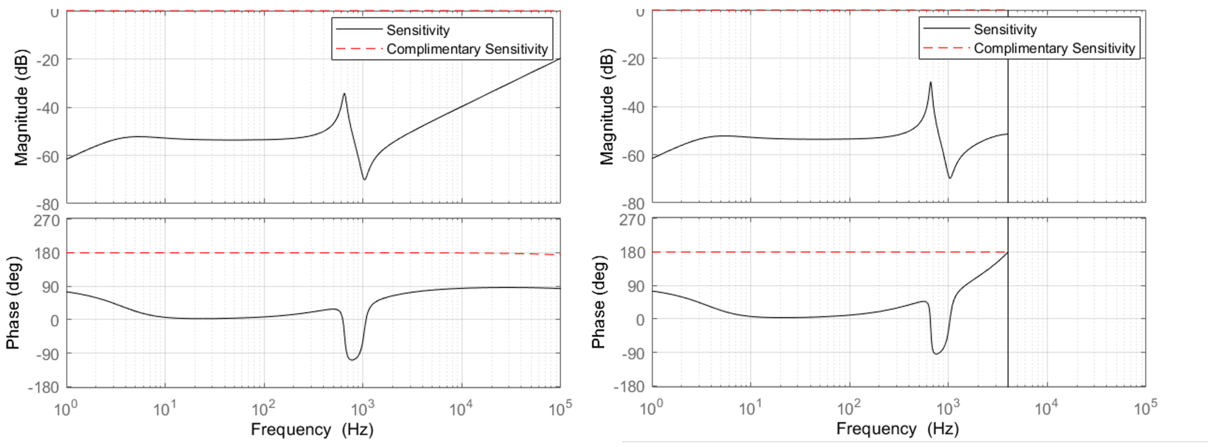

Figure 20: Continuous (L) vs Discrete Sensitivity

3.6 Question 14: Feedforward Controller

With no control, the plant does not follow the reference signal. This is quite predictable as the reference signal is a displacement whereas the input to the plant is a force.

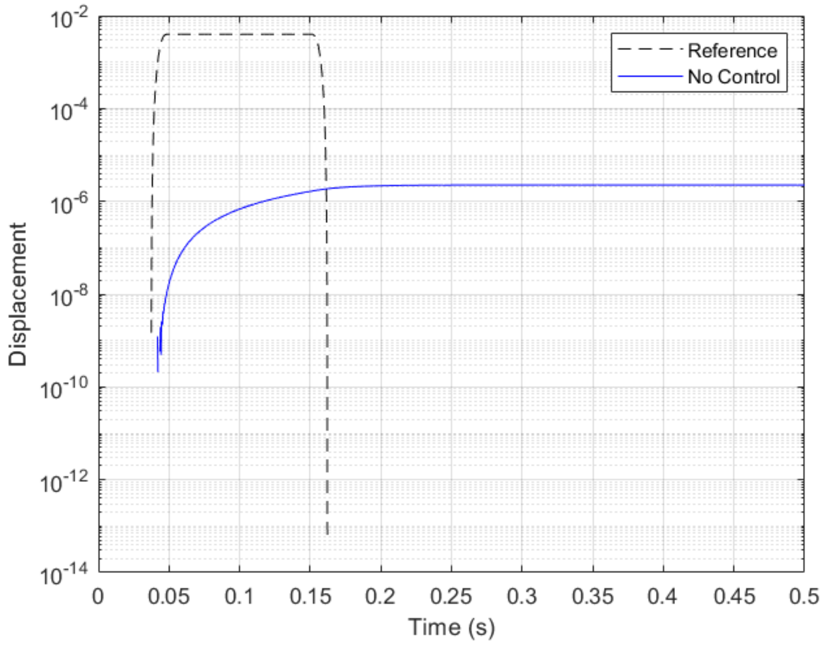

Figure 21: Reponse with no control

Implementing a feedforward controller boosts the reference signal so the response mirrors the reference more closely. This reduces the error the PID controller has to deal with, making it more accurate.

It was designed to generate a force equivalent to accelerating the mass and acting against the damper. The mass force requires the displacement input to be differentiated twice and has a gain of 5 (Equation 20), equivalent to the sum of the middle and top mass (Table 1). The damping force is only differentiated once. The gain, c_{eq} was initially calculated as 167 as the middle and top damper in series (Equation 21), but was revised to 185 as it had a better response with the PID controller (Figure 30).

$$F = ((m_2 + m_3)s^2 + c_{eq}s)\hat{x}_s \tag{20}$$

$$c_{eq} = \frac{1}{\frac{1}{c_2} + \frac{1}{c_3}} \tag{21}$$

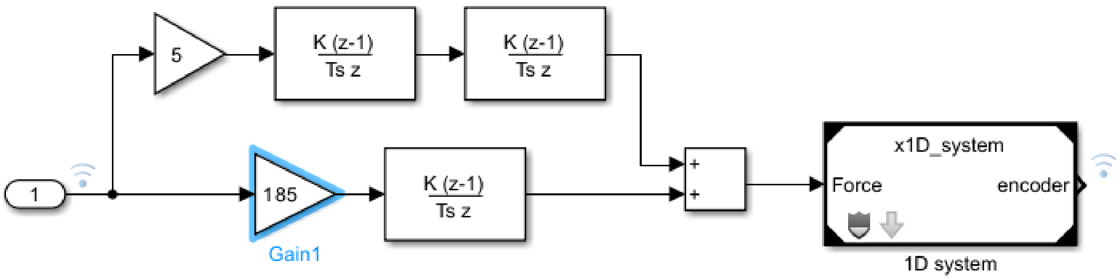

Figure 22: Feedforward Diagram

The contributions from the mass and damping sections of the feedforward controller can be seen in (Figure 23)

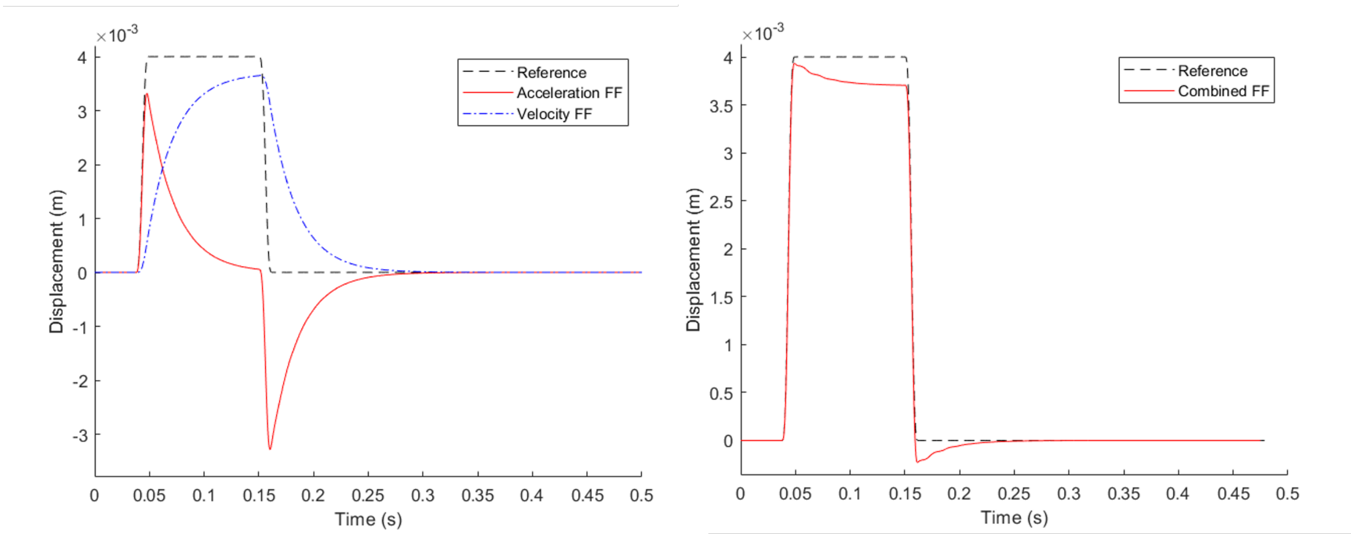

Figure 23: Feedforward Controller

This was then combined with the PID feedback controller (Figure 24). The response to the reference signal can be seen in (Figure 25)

The combined controller is clearly superior (Table 4) with the lowest overshoot (0.42%) for the step up and the lowest undershoot for the step-down (0.32 %). Since the combined controller did not exceed 2% deviation from the reference, the settling time was defined as the time to reach ±0.2% of the reference signal after the step down. The rise time was defined as the lag time between the reference and response reaching the 4mm displacement.

Table 4: Controller Time Domain Parameters

	Feedforward	**Feedback**	**Combined**
Overshoot (%)	N/A	2.2	0.42
Undershoot (%)	5.7	1.9	0.32
Settling Time (ms)	84.85	164.7	28.3
Rise Time (ms)	N/A	2.835	5.61

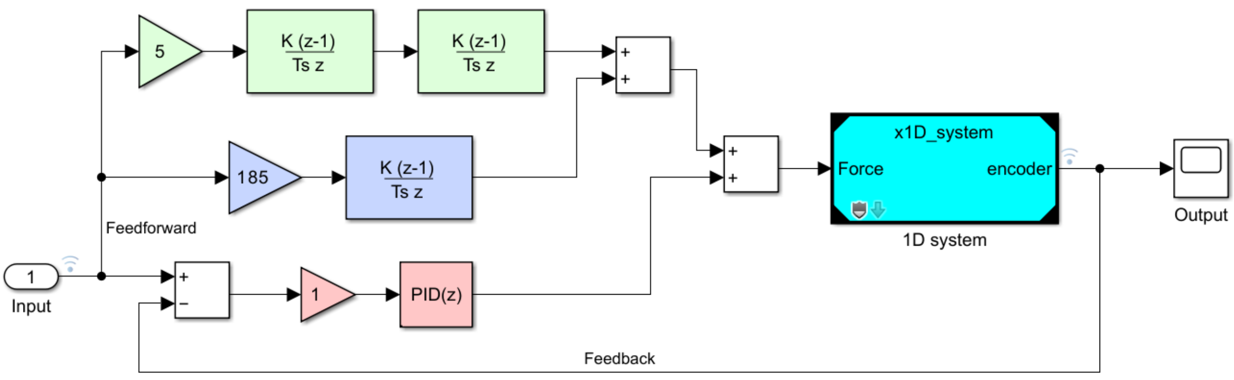

Figure 24: Feedback(red) and Feedforward (green and blue) Controller Diagram

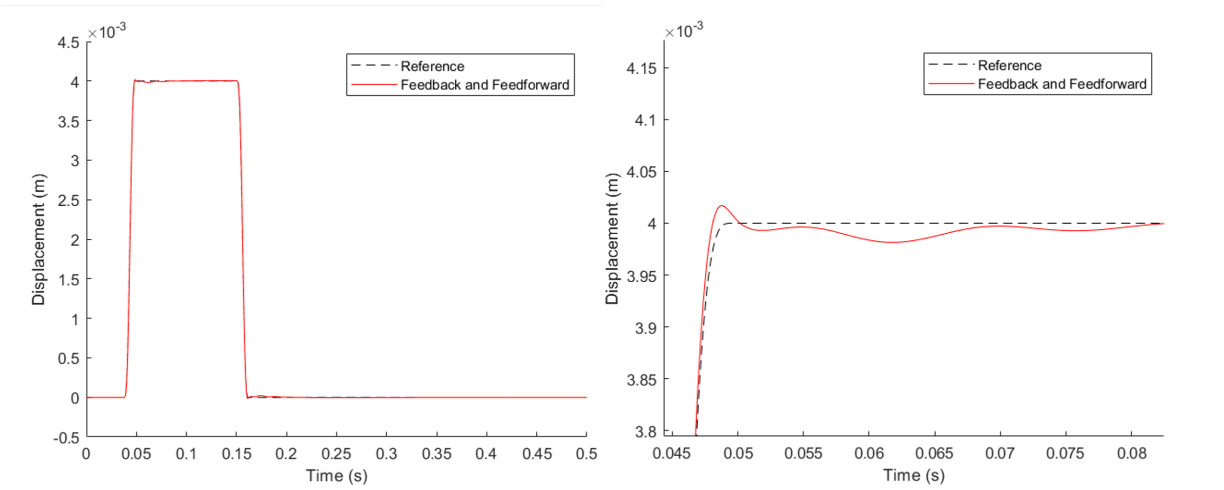

Figure 25: Feedback and Feedforward Controller: Overall (L) and Close-up (R)

This can be seen in (Figure 26).

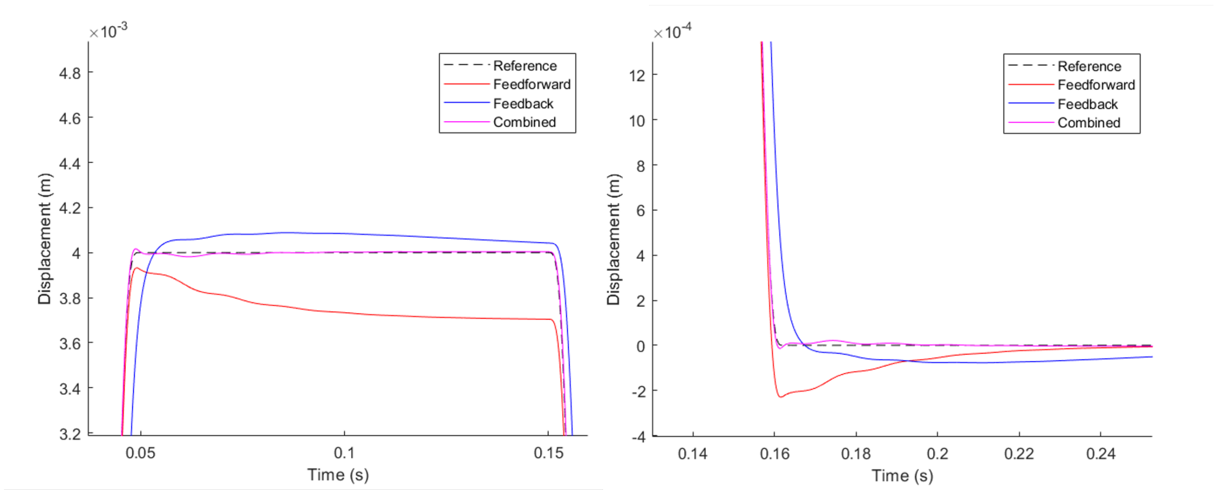

Figure 26: Comparing Controllers: Step-up (L), Step-down (R)

3.7 Question 15: Disturbance Rejection

The error (e_d) caused by a disturbance (d) can be described by (Equation 22)

$$e_d = P(s)d = \frac{G}{1+CG}d \tag{22}$$

Where P is the process sensitivity. The combined controller performs better than the feedback controller at frequencies above 100Hz

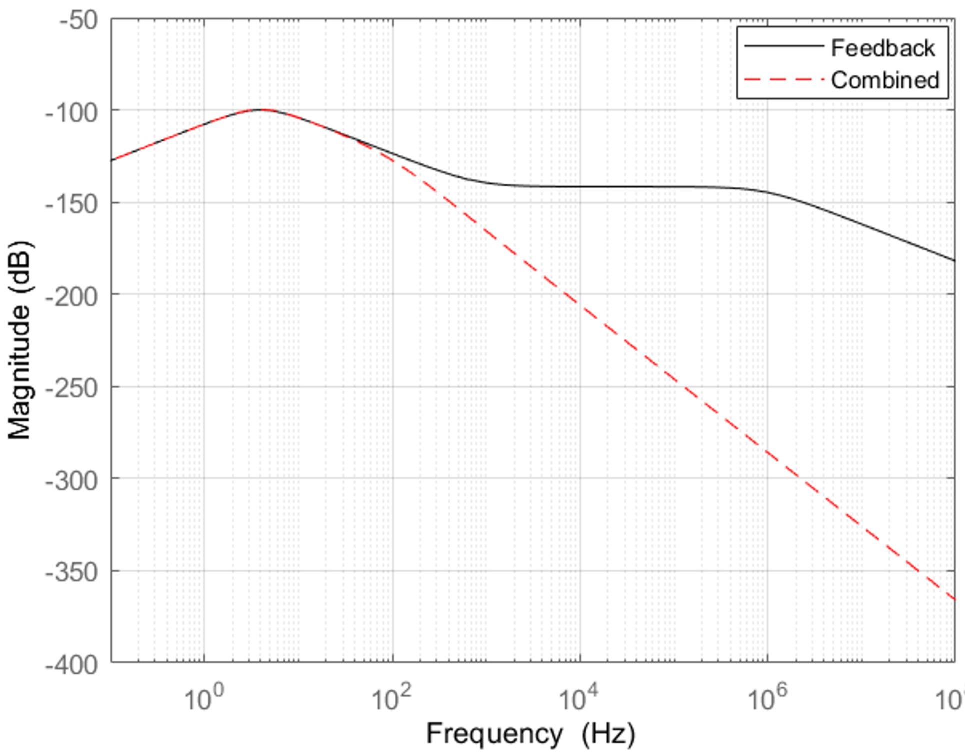

Figure 27: Process Sensitivity

The maximum force required should also be considered when choosing the actuator to control the system. For the system with no disturbances or noise, (Figure 28) the maximum force required is 1120N.

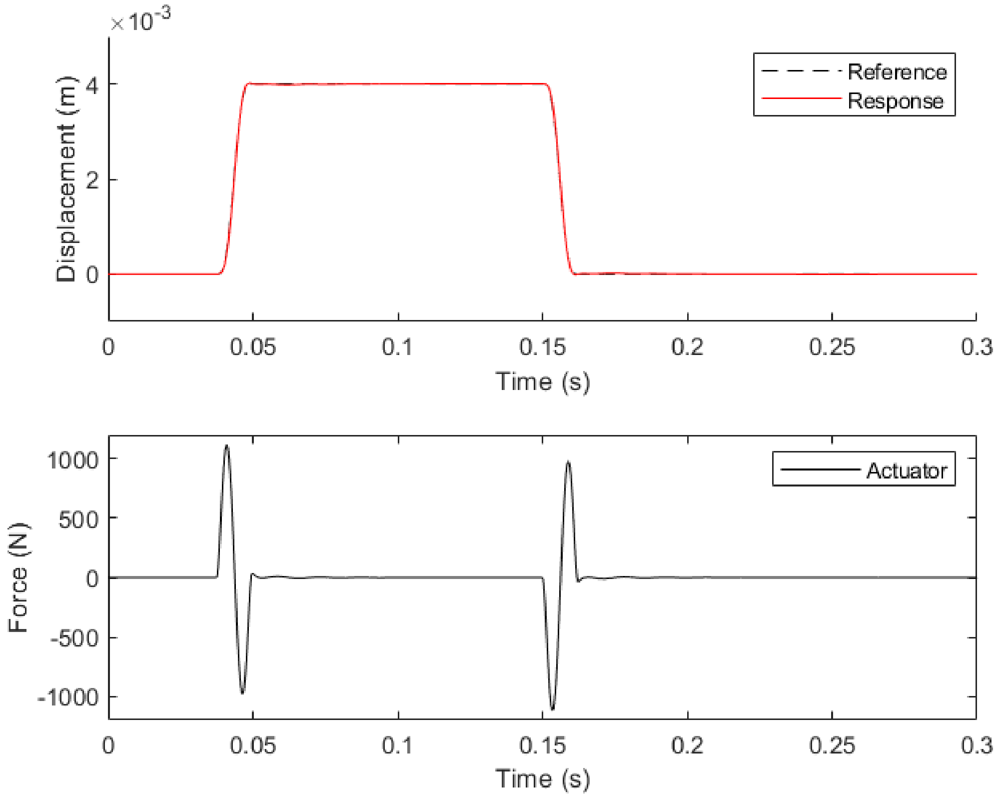

Figure 28: Actuator Force (no noise or disturbances)

Noise from the encoder up to the order of $\pm 10\mu m$ is allowable to ensure noise does not dominate (Figure 29)

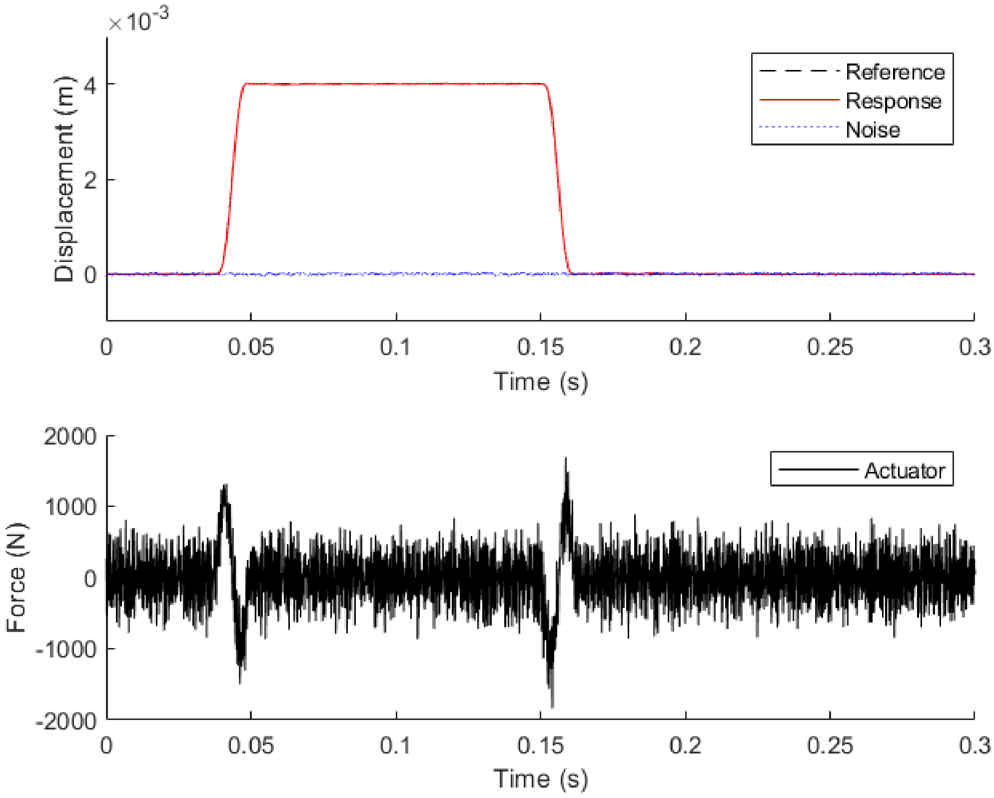

Figure 29: Actuator Force with noise $\pm 10\mu m$

3.8 Question 16: Highest Theoretical Bandwidth

The highest bandwidth without delay was calculated to be the same as with delay, at 74.9 Hz. However, this is unrealistic as without delay, the bandwidth should be significantly higher.

Appendix

1: Algebraic Transfer functions

$x_1(s) = ((-m2*m3*laplace(f(t),t,s))*s^3+(-c3*m2*laplace(f(t),t,s)-c3*m3*laplace(f(t),t,s))*s^2+(-k2*m2*laplace(f(t),t,s)-k2*m3*laplace(f(t),t,s))*s)/(m1*m2*m3*s^5+(c1*m2*m3+c2*m1*m3+c3*m1*m2+c2*m2*m3+c3*m1*m3)*s^4+(c1*c2*m3+c1*c3*m2+c2*c3*m1+c1*c3*m3+c2*c3*m2+c2*c3*m3+k2*m1*m2+k1*m2*m3+k2*m1*m3)*s^3+(c1*c2*c3+c1*k2*m2+c2*k2*m1+c1*k2*m3+c2*k1*m3+c2*k2*m2+c3*k1*m2+c2*k2*m3+c3*k1*m3)*s^2+(c1*c2*k2+c2*c3*k1+k1*k2*m2+k1*k2*m3)*s+c2*k1*k2)$

$x_2(s) = (m1*m3*laplace(f(t),t,s)*s^4+(c1*m3*laplace(f(t),t,s)+c3*m1*laplace(f(t),t,s))*s^3+(c1*c3*laplace(f(t),t,s)+k2*m1*laplace(f(t),t,s)+k1*m3*laplace(f(t),t,s))*s^2+(c1*k2*laplace(f(t),t,s)+c3*k1*laplace(f(t),t,s))*s+k1*k2*laplace(f(t),t,s))/(m1*m2*m3*s^6+(c1*m2*m3+c2*m1*m3+c3*m1*m2+c2*m2*m3+c3*m1*m3)*s^5+(c1*c2*m3+c1*c3*m2+c2*c3*m1+c1*c3*m3+c2*c3*m2+c2*c3*m3+k2*m1*m2+k1*m2*m3+k2*m1*m3)*s^4+(c1*c2*c3+c1*k2*m2+c2*k2*m1+c1*k2*m3+c2*k1*m3+c2*k2*m2+c3*k1*m2+c2*k2*m3+c3*k1*m3)*s^3+(c1*c2*k2+c2*c3*k1+k1*k2*m2+k1*k2*m3)*s^2+c2*k1*k2*s)$

$x_3(s) = (c3*m1*laplace(f(t),t,s)*s^3+(c1*c3*laplace(f(t),t,s)+k2*m1*laplace(f(t),t,s))*s^2+(c1*k2*laplace(f(t),t,s)+c3*k1*laplace(f(t),t,s))*s+k1*k2*laplace(f(t),t,s))/(m1*m2*m3*s^6+(c1*m2*m3+c2*m1*m3+c3*m1*m2+c2*m2*m3+c3*m1*m3)*s^5+(c1*c2*m3+c1*c3*m2+c2*c3*m1+c1*c3*m3+c2*c3*m2+c2*c3*m3+k2*m1*m2+k1*m2*m3+k2*m1*m3)*s^4+(c1*c2*c3+c1*k2*m2+c2*k2*m1+c1*k2*m3+c2*k1*m3+c2*k2*m2+c3*k1*m2+c2*k2*m3+c3*k1*m3)*s^3+(c1*c2*k2+c2*c3*k1+k1*k2*m2+k1*k2*m3)*s^2+c2*k1*k2*s)$

2: Feedforward and Feedback Controller

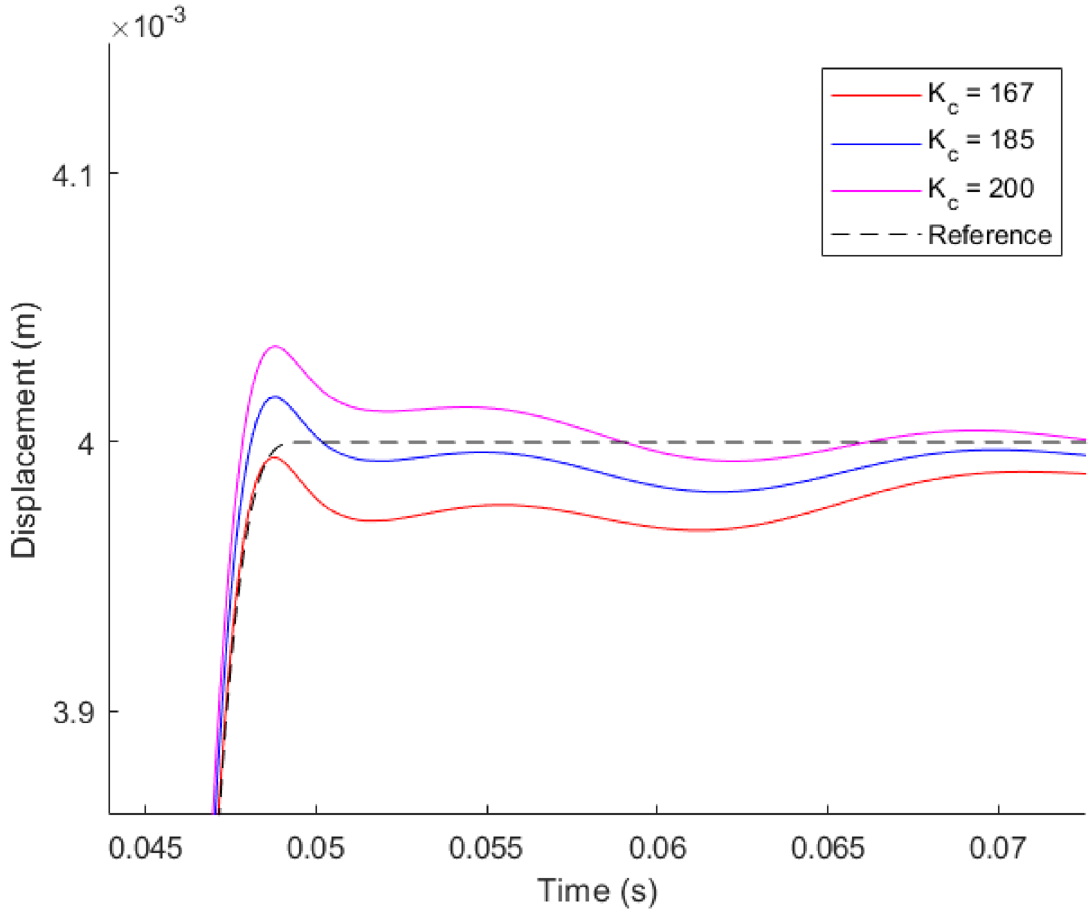

Figure 30: Feedforward Diagram Damping

References