Assignment 2

ME46085 Mechatronic System Design

by

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1. Section 1: System Identification

1.1 Question 1: Explain why the closed-loop identification procedure was used in this case

Close loop identification is preferred to open loop identification if the system and controller have already been created because it is more convenient to measure the entire system instead of just the plant. Also, the plant may be unsteady without closed loop control making open loop identification impossible. Finally, if there is noise after the plant, closed loop identification can help to reject this noise.

1.2 Question 2: Plot the provided signals in the time domain. What can be interpreted from the characteristics of input disturbance signal f and position output x?

The first 12 seconds of 100 seconds of data is shown in (Figure 1). The input disturbance (f) is a chirp signal repeating every 10 seconds. From f to x, the amplitude of x steadily increases as the frequency of the chirp signal increases, and then beyond the peak at 4 seconds it quickly returns to zero. This suggests that the disturbance (f) frequency at 4 seconds is a resonant frequency for x. Also, at very high frequencies, the output suddently spikes, suggesting a 2nd resonant point. The error is simply the output multiplied by -1, as the reference signal is 0.

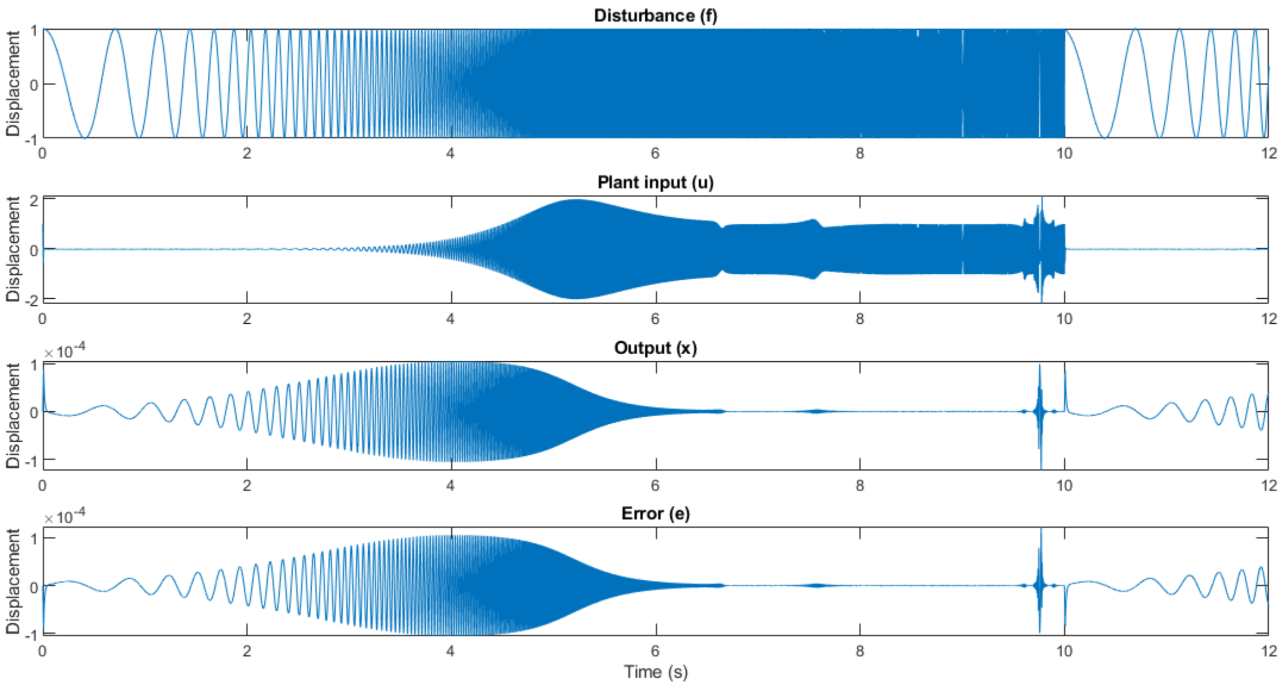

Figure 1: Time Domain Signals

1.3 Question 3: Calculate and present the closed-loop frequency responses from input disturbance f to u, and x, respectively. Explain what these frequency response functions represent in the closed-loop system.

The two transfer functions u/f and x/f were determined using tfestimate in MATLAB without any window.

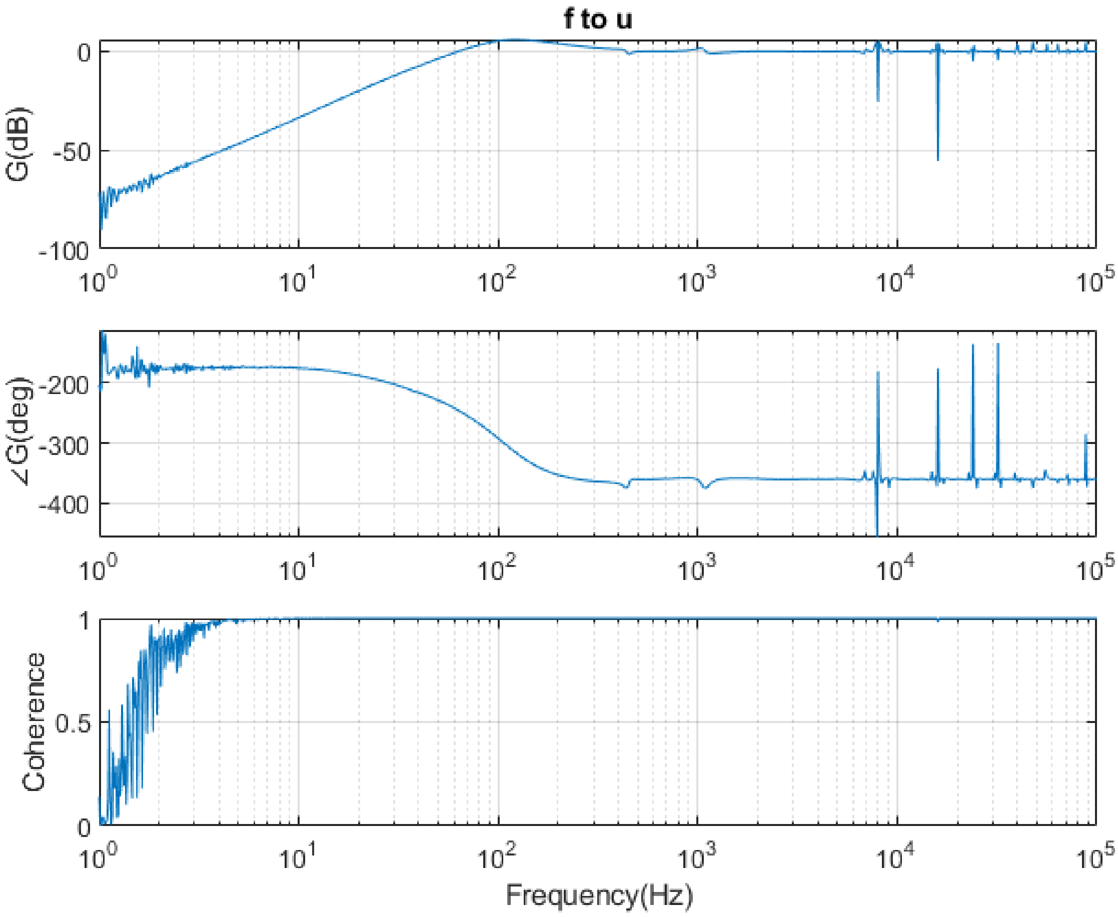

Figure 2: Bode Plot of F to U

The f to u transfer function (Figure 2) has a low coherence at frequencies below 10Hz. It has a +2 gradient until 100Hz, where is has a 0 gradient. At low frequencies, the phase approaches -180° and at high frequencies it approaches -360°. This represents the impact of the disturbance on the input into the plant.

The f to x transfer function (Figure 3) on the other hand has a low coherence above 500Hz. At low frequencies, the phase approaches 90°. This represents the impact of disturbances on the system on the position output.

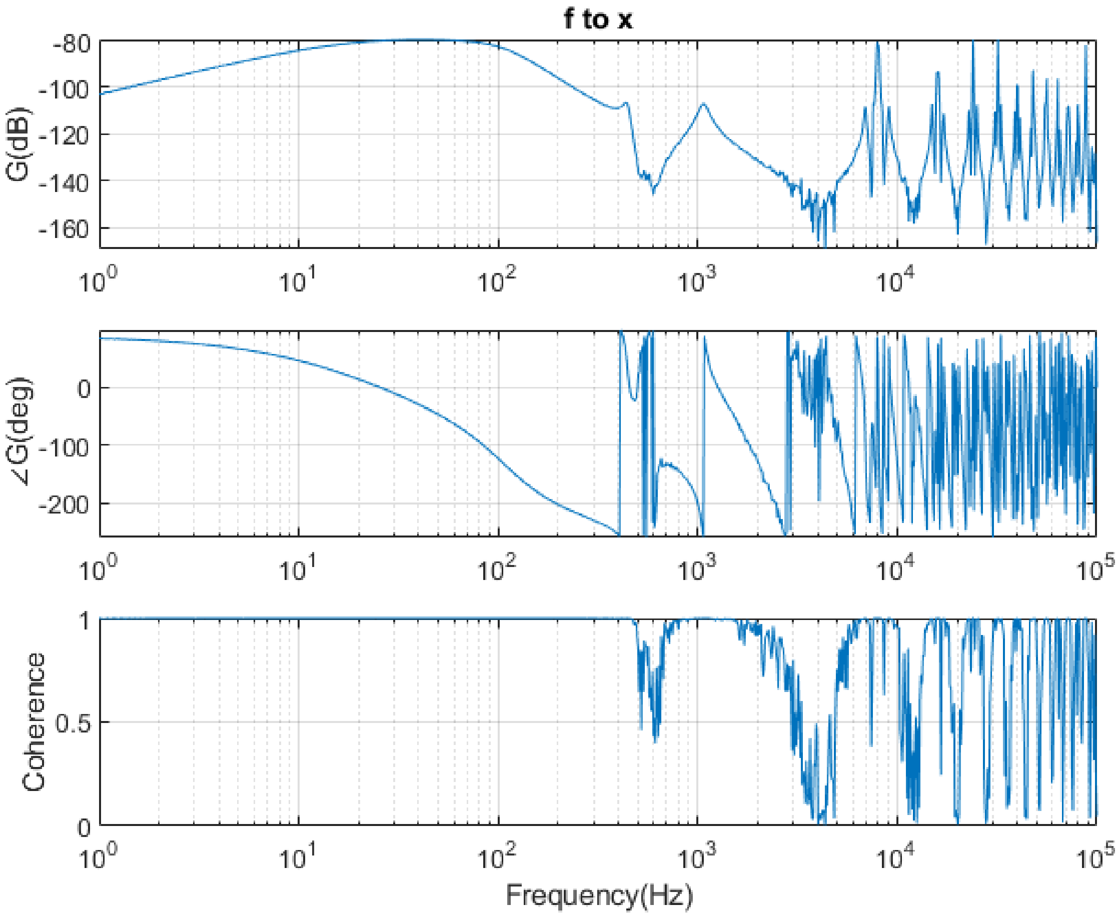

Figure 3: Bode Plot of F to X

1.4 Question 4: Calculate and present the frequency response of the plant. Explain analytically the procedure used to obtain the plant model using the results from Q3. Further comment and give reasons for the possible differences from the mathematical plant obtained in Assignment Part 1.

The Bode plot f to u, represents the sensitivity function (Equation 1) and f to x represents the process sensitivity function (Equation 2).

$$\frac{\hat{u}}{\hat{f}} = \frac{1}{1+CP} = S \tag{1}$$

$$\frac{\hat{x}}{\hat{f}} = \frac{P}{1+CP} = PS \tag{2}$$

Therefore, dividing the process sensitivity function by the sensitivity function gives the plant transfer function P (Figure 4)

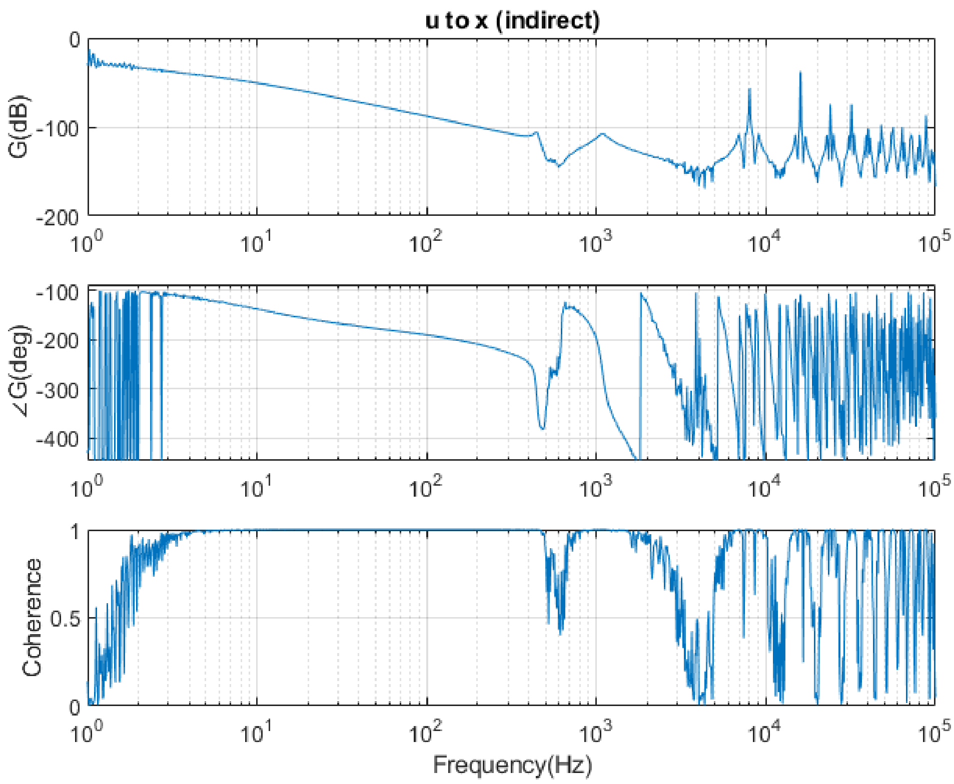

Figure 4: Bode Plot of U to X (indirect)

The coherence of the u to x transfer function was calculated by multiplying the corresponding coherences of the sensitivity and closed loop transfer function.

The magnitude of the closed loop identification was 100 times lower than the open loop calculated in Assignment 1 (Figure 5), so in the comparison it has been multiplied by 100. Both methods of identification show a resonant peak just above 1000Hz and an anti-resonance around 600Hz. However, since the coherence decreases significantly around 600Hz, the anti-resonance is less accurate and well-defined compared to the theoretical transfer function. At frequencies between 5 and 100Hz, the closed loop identification matches the mathematical functions closely, however above 100Hz the phases do not match. This may be due to sensor delay in the system, as the phase at high frequencies matches the delayed transfer function calculated in Assignment 1 more closely (Appendix 1: Figure 15). Also, the magnitude at high frequencies matches those of the experimentally determined transfer function in Assignment 1 (Appendix 1: Figure 16).

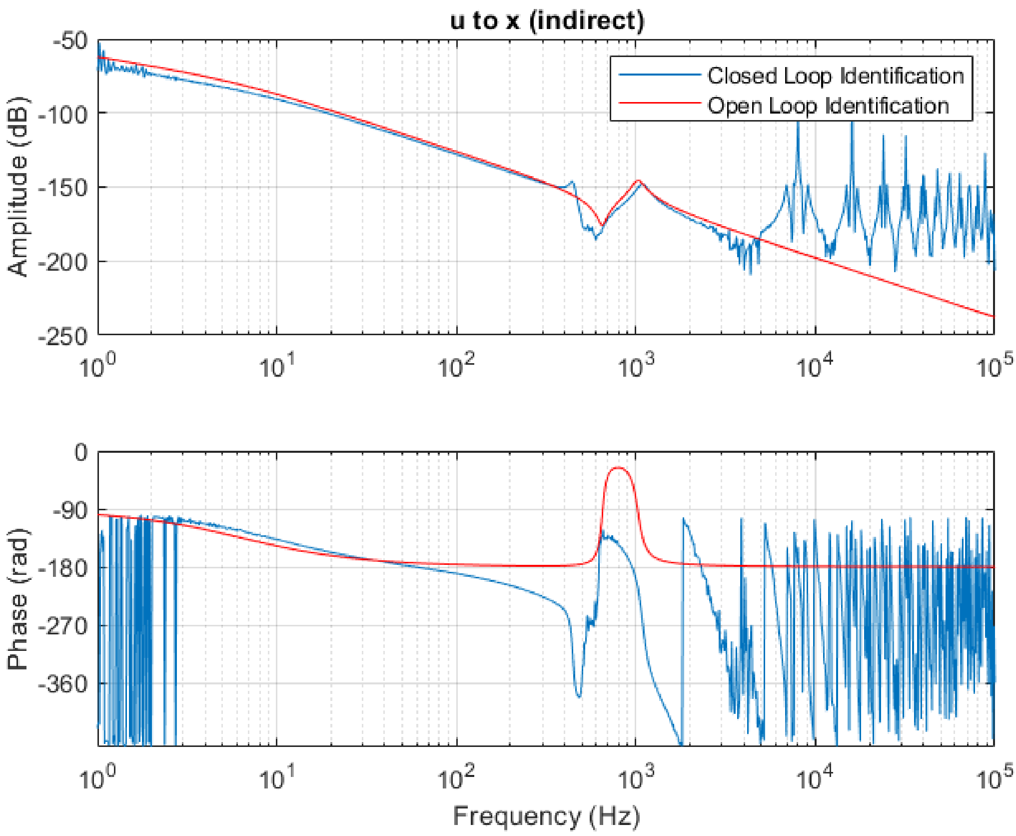

Figure 5: Comparison of Open Loop (Assignment 1) and Closed Loop Identification

1.5 Question 5: Compute the transfer function from u to x, to directly estimate the plant model and compare your results with results obtained in Q4. Comment if there are observed differences.

Instead of calculating the sensitivity and closed loop transfer function, the plant transfer function was directly calculated (Figure 6). Direct identification is almost identical, except at freqencies below 1Hz, however this is probably irrelevant since the coherence below 1Hz is generally below 0.5.

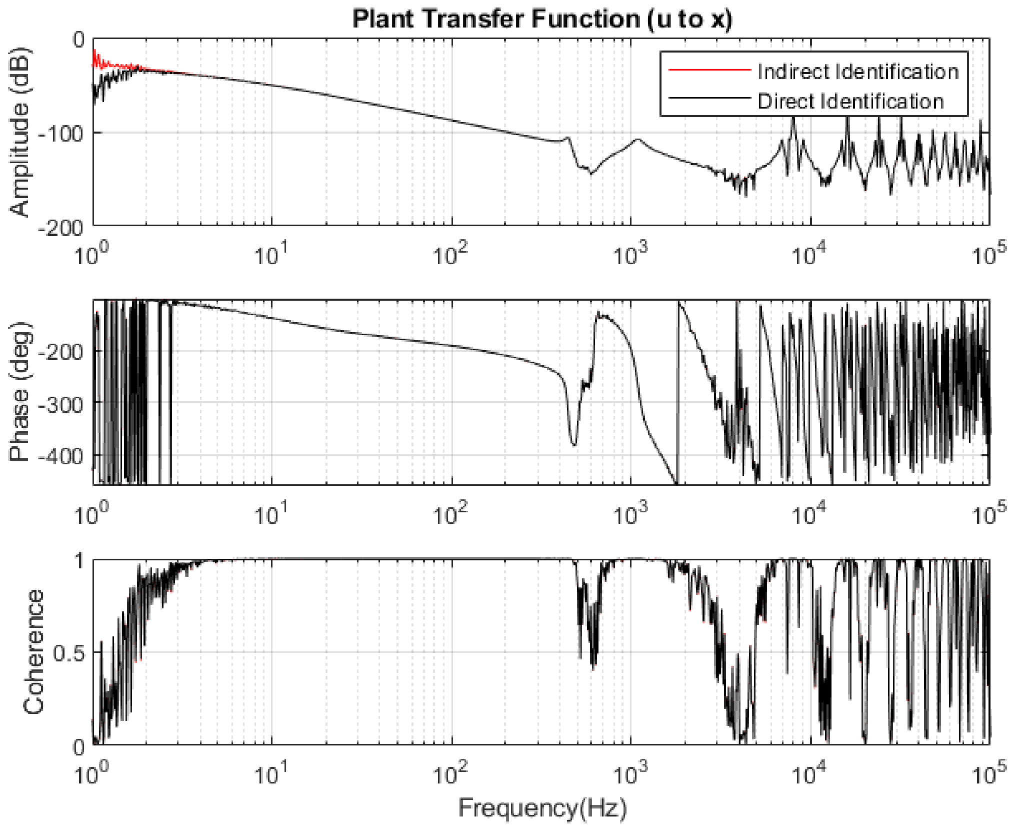

Figure 6: Bode Plot of U to X (direct)

2. Section 2: Controller Design

2.1 Question 6: Calculate and present the frequency responses of the controller (C) used in the experiments

The controller output $v = u - f$ (Figure 7 & 8) was compared to the error (e) (Figure 1).

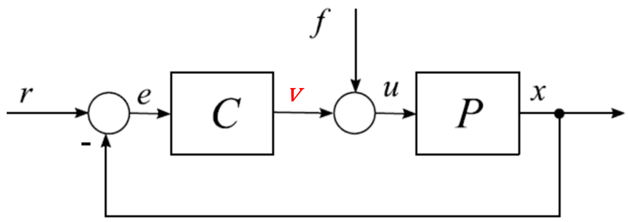

Figure 7: Block Diagram 2

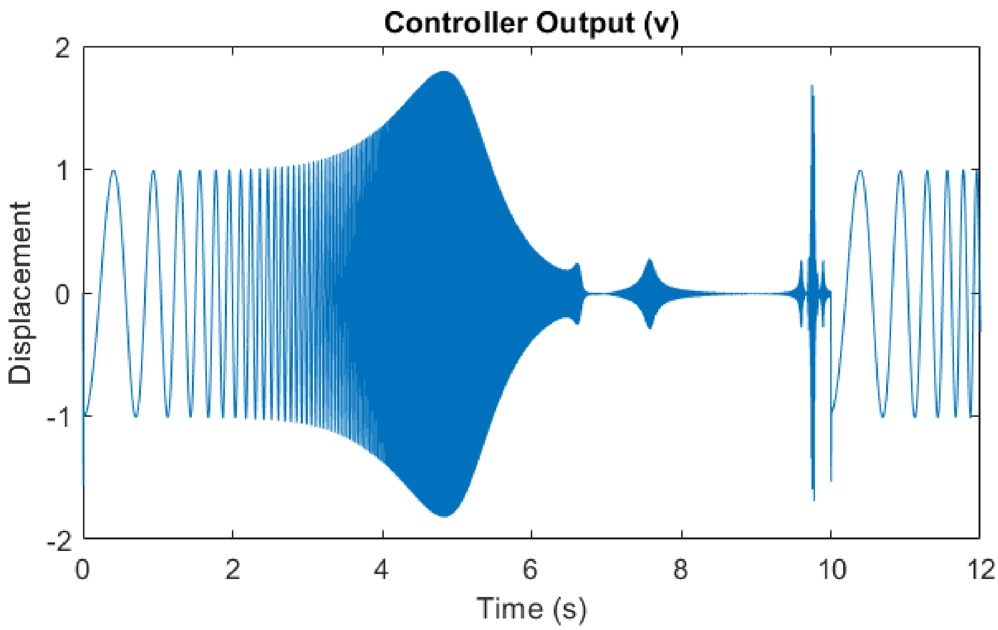

Figure 8: Controller Output (v)

The controller transfer function $\frac{v}{e}$ (Figure 9) was again calculated using tfestimate in MATLAB.

2.2 Question 7: recreate the controller in MATLAB based on the controller estimated in Q6. Present the used transfer functions and controller parameters. Compare the frequency response of the recreated controllers with the experimental results of Q6

The controller transfer function appears to be a tamed PID controller (Equation 3).

$$C = P + \frac{I}{s} + \frac{ND}{1 + \frac{N}{s}} \tag{3}$$

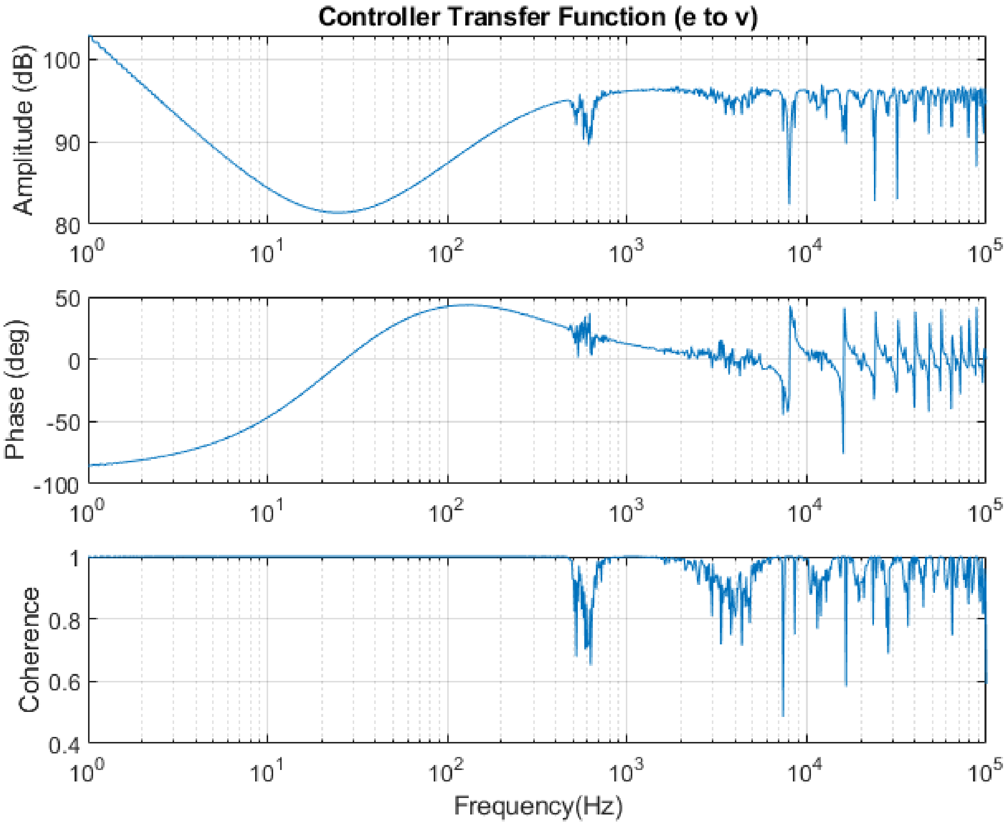

Figure 9: Transfer Function of Controller

Substituting $s = jw$ into (Equation 3) and rearranging gives (Equation 4)

$$Re_c(w) = P + \frac{ND\omega^2}{N^2 + \omega^2}, \quad Im_c(w) = -\frac{I}{\omega} + \frac{DN^2\omega^2}{N^2 + \omega^2} \tag{4}$$

Using the magnitude of a complex number $|x| = \sqrt{Re^2 + Im^2}$, the amplitude of the controller can be re-written as (Equation 5)

$$|C(\omega)| = \sqrt{P^2 + \frac{DN((2P + DN)\omega^2 - 2IN)}{(N^2 + \omega^2)} + \frac{I^2}{\omega^2}}; \tag{5}$$

It can then be shown that $lim_{\omega \to \infty}(|C(w)|) = P + ND$. Using brute force the parameters of the controller were calculated.

Table 1: Mass, damping and stiffness variables

Parameter	**Symbol**	**Value**
Proportional Gain	P	1.1239e+04
Integral Gain	I	1.3977e+05
Derivative Gain	D	200.00
Filter Coefficient	N	275.00

The theoretical controller closely matches the actual controller below 1100Hz except for a trough at 400Hz. This may be due to an additional notch filter, however the coherence of the experimental controller also decreases at 400Hz so due to this uncertainty, no additional filters were added to the controller. (Equation 6).

$$\angle C(\omega) = \arctan(\frac{DN^2\omega - I\frac{N^2+\omega^2}{\omega}}{P(N^2+\omega^2) + DN\omega^2}); \tag{6}$$

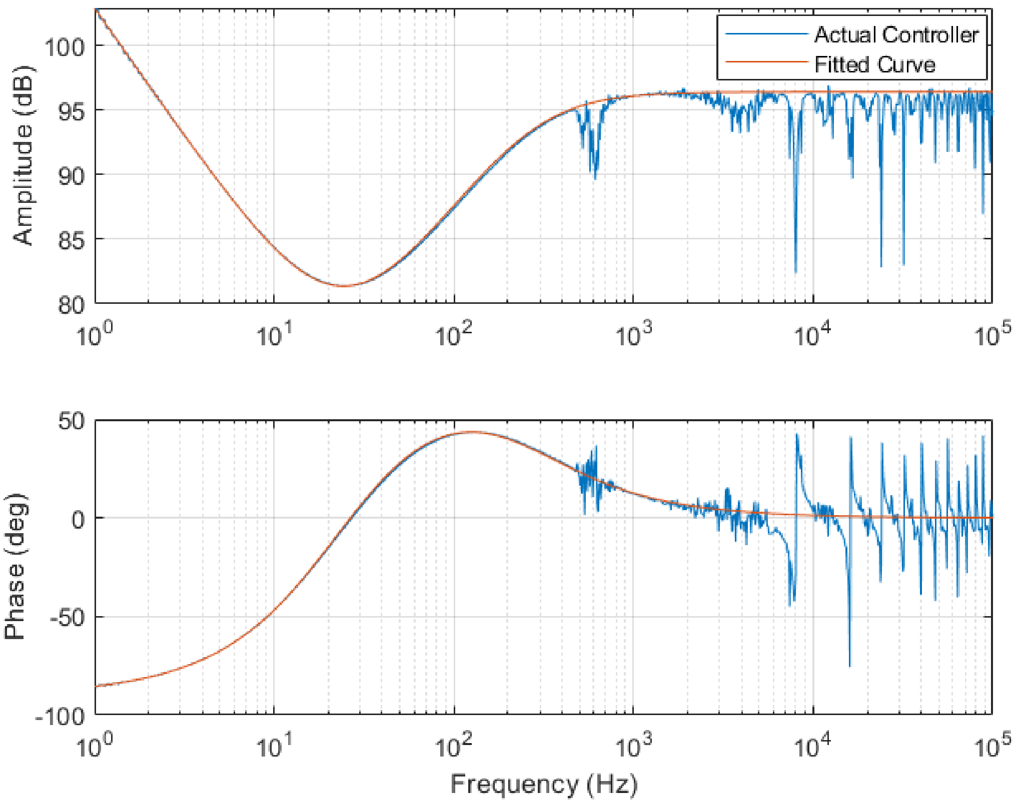

Figure 10: Experimental vs Theoretical (Fitted Curve) Controller

Overall, the average error is 0.5% (Figure 11)

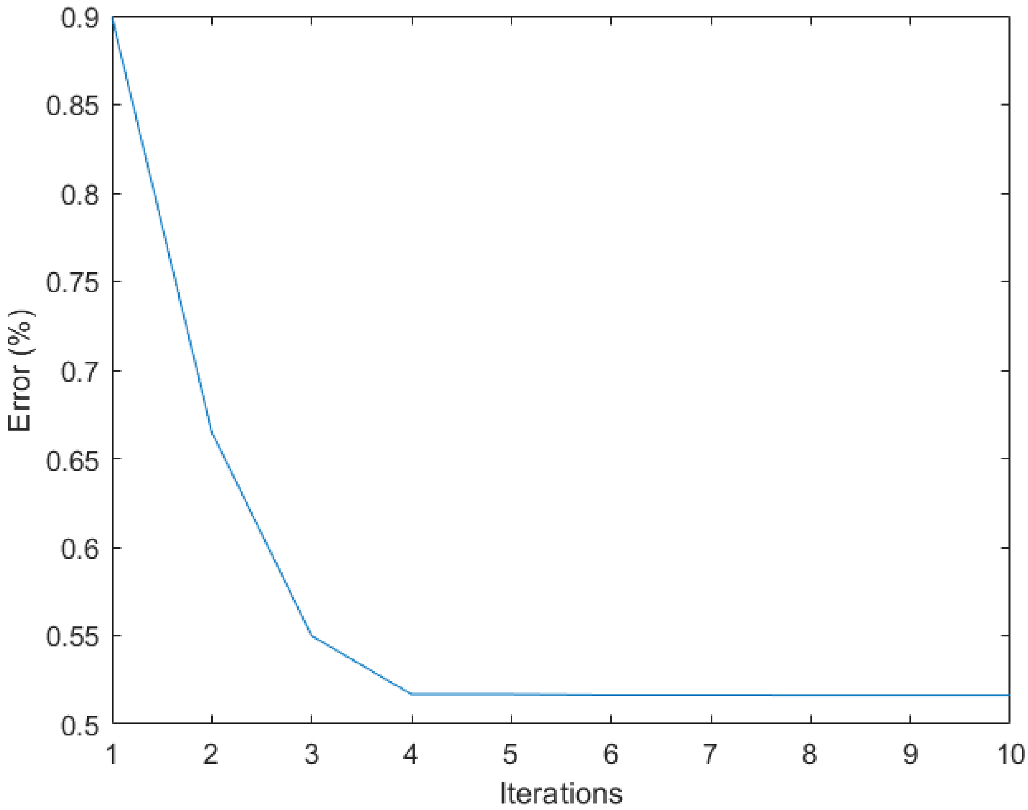

Figure 11: Error of Theoretical Controller

2.3 Question 8: Re-create all the closed-loop transfer functions considered in this assignment using the recreated controller and estimated model of the plant. Compare the recreated and measured closed-loop transfer functions. Comment on the differences

The closed loop transfer function (Equation 7) was determined for the measured controller (Figure 9) and the recreated controller (Figure 10) using the indirectly estimated plant (Figure 4).

$$G_{CL} = \frac{GC}{1+GC}; \tag{7}$$

There is very little difference between the measured and re-created controller (Figure 12), except there is a slight depression in the magnitude of the recreated closed loop at 30 to 100Hz. This is expected since the recreated controller was very similar to the measured controller (Figure 10).

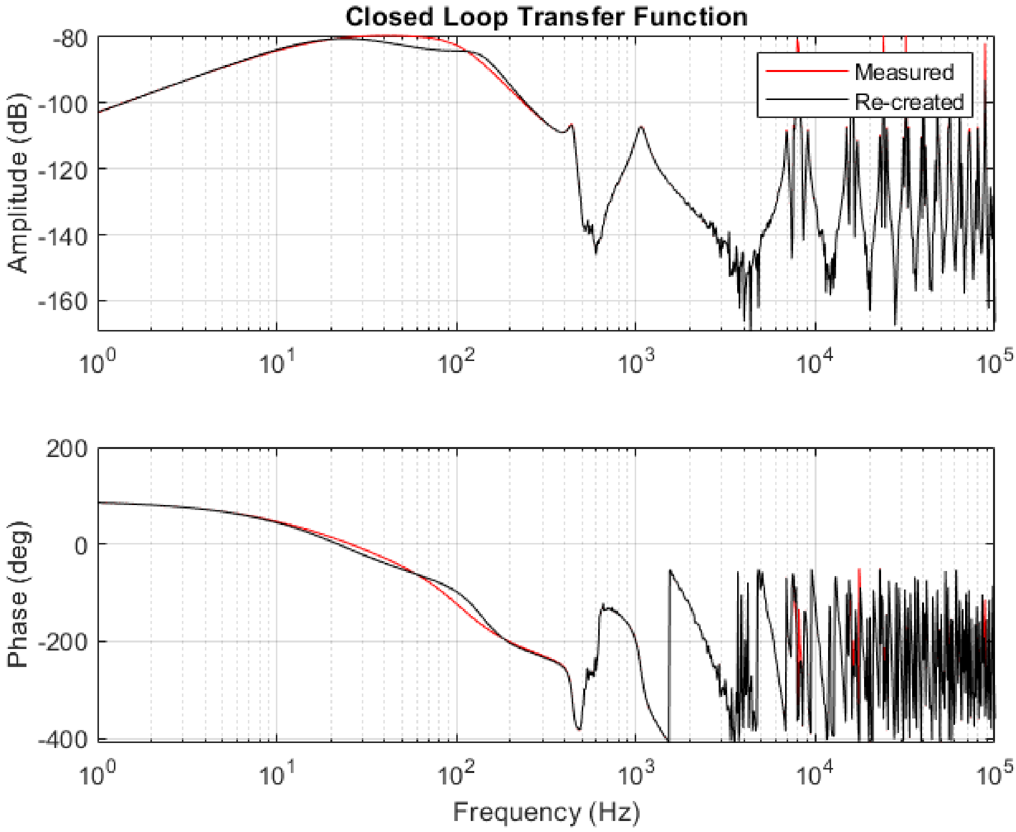

Figure 12: Closed Loop Transfer Function

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2.4 Question 9: Present the closed-loop response to a step reference input (r). For this case, don't include the disturbance signal (f) in the closed loop. Use the recreated feedback controller (Q7) and estimated plant (Q4)

Initially, tfest was used to approximate the transfer function from question 4. Unfortunately it gave an erroneous result (Appendix 2).

Instead, adaptations to the mathematical plant from assignment 1 were implemented. Because of the high frequency, the error was calculated for frequencies below a cut-off frequency (Figure 13).

The parameters for the original plant were adapted (Equation 8) to Table 2, to create an approximation to the plant that could be mathematically implemented.

$$G_p = e^{-ts}\frac{A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4}{B_0 + B_1 s + B_2 s^2 + B_3 s^3 + B_4 s^4 + B_5 s^5 + B_6 s^6}; \tag{8}$$

Then, to copy the drop in phase from 100 to 400Hz, a time delay of 0.125ms was added to the plant.

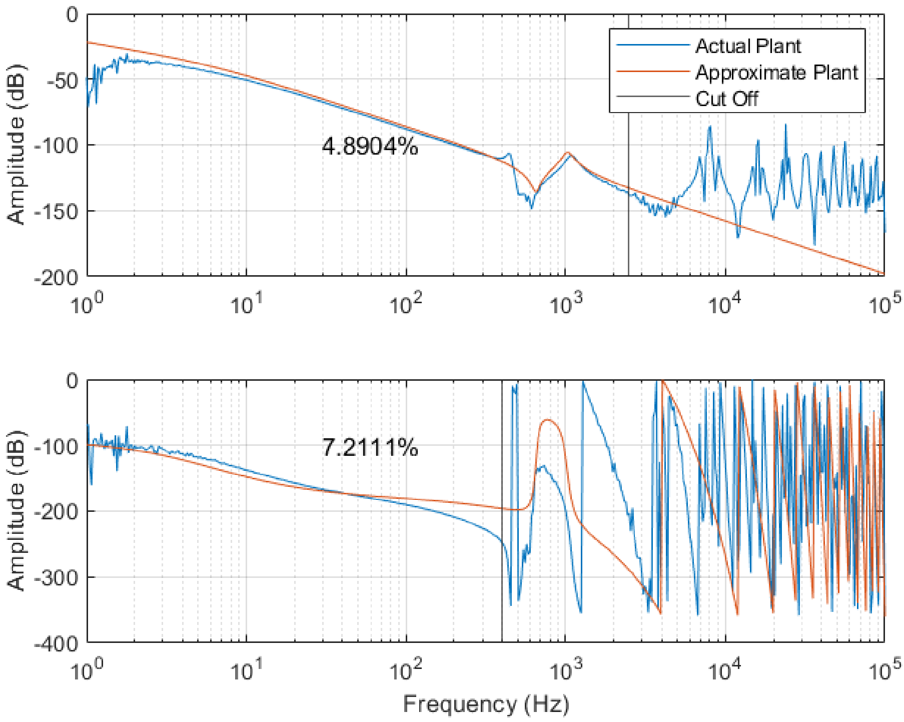

Figure 13: Comparison of the measured and approximate analytical plant

Table 2: Plant Transfer Function Parameters

Parameter	**Value**	**Parameter**	**Value**
A0	4e15	**B1**	1.6e18
A1	2.5e12	**B2**	26e15
A2	20e9	**B3**	24e12
A3	550e3	**B4**	124e9
A4	1.1e3	**B5**	4.1e6
B0	0	**B6**	3e3

Overall, it is a satisfactory approximation, with an average error of 5% and 7% for the magnitude and phase respectively, however it could be improved. The error is mainly caused by the assumption to neglect the effect of parasitic dynamics around 500Hz.

However, when implementing this transfer function into Simulink, the output became unstable and the displacements tended towards infinity. This also occurred when using the mathematical plant from assignment 1, which suggests I made the Simulink block incorrectly, instead of the plant being wrong. Therefore to calculate the step response, I used the hidden plant from assignment 1 (Figure 14). The controller provides an underdamped response with a maximum overshoot of 16 %.

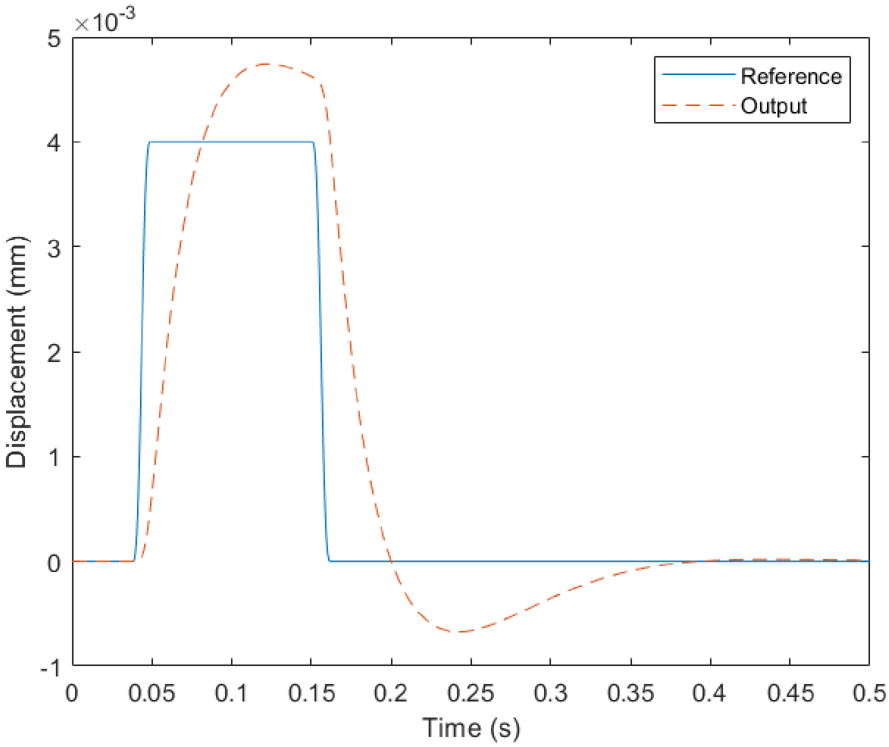

Figure 14: PID control vs reference signal

Appendices

Appendix 1: Plant Comparison

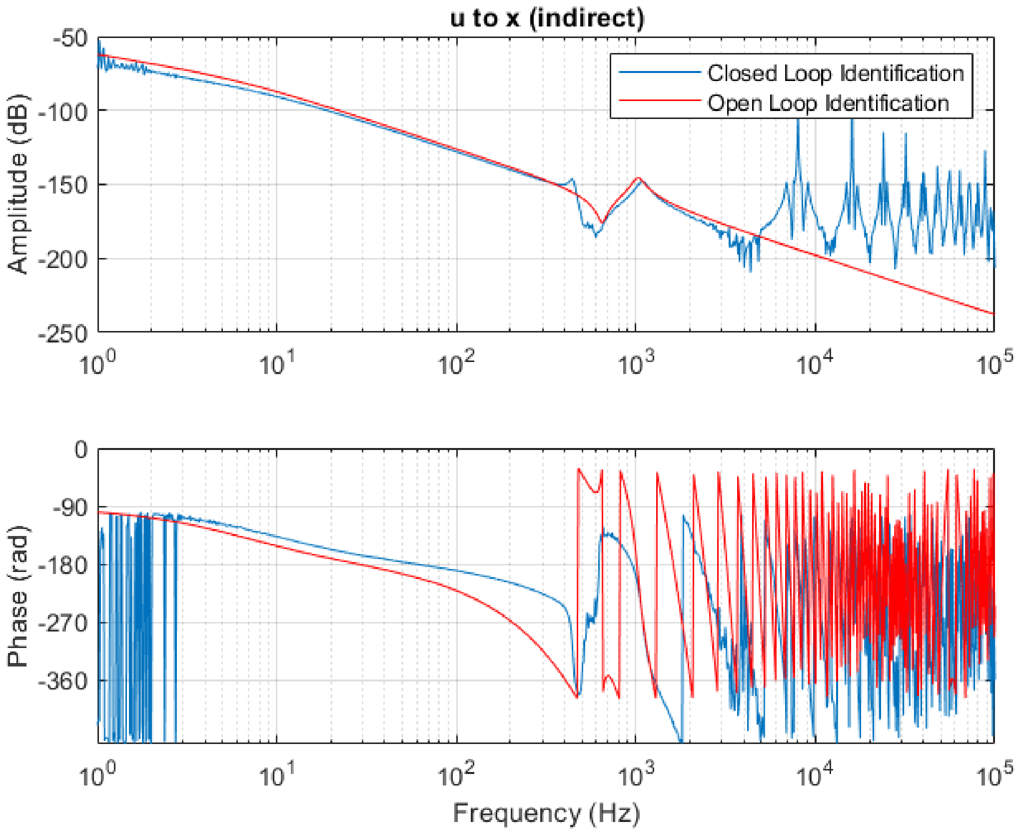

Figure 15: Comparison of Delayed Open Loop (Assignment 1) and Closed Loop Identification

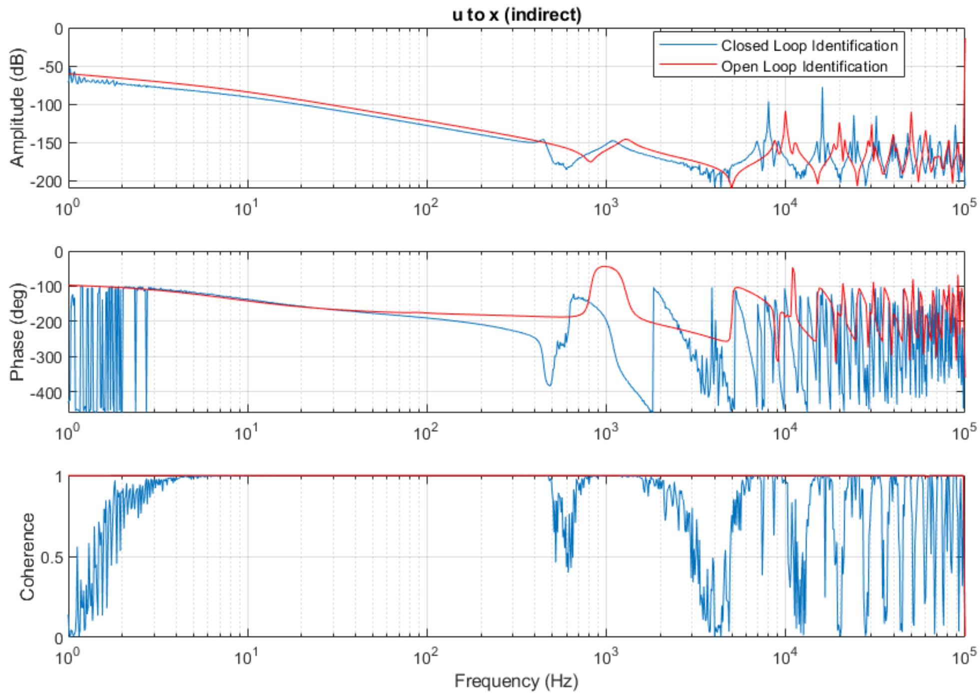

Figure 16: Comparison of Experimental Open Loop (Assignment 1) and Closed Loop Identification

Appendix 2: Approximate Plant from tfest

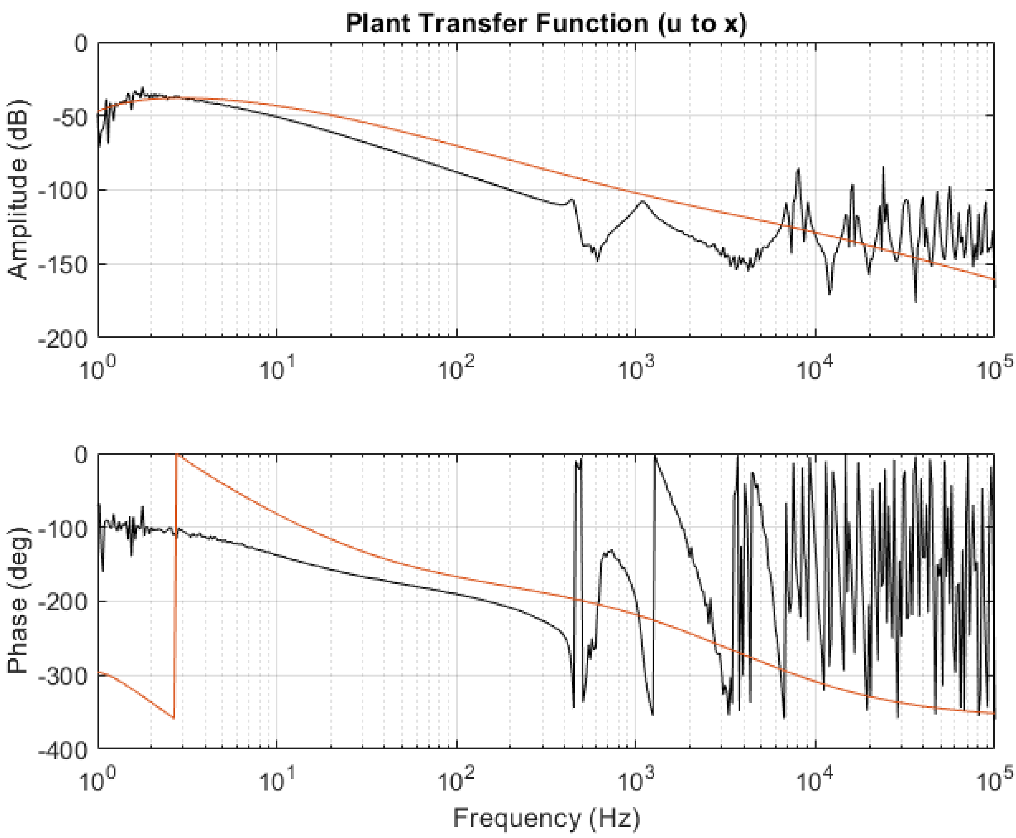

Figure 17: Approximate Plant (tfest)

References