Assignment 1: Anti-lock Braking (ABS) Controller Design

RO47017 Vehicle Dynamics and Control

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Contents

1. Introduction

Anti-lock braking (ABS), is a control system which aims to ensure the driver maintains control of the vehicle during emergency braking on slippery road conditions. It is achieves this by preventing the wheels "locking up", defined as when the wheel slip (κ) is one. During deceleration this is defined as:

$$\kappa = \frac{V_x - \omega r}{V_x} \tag{1}$$

Once a wheel locks up, the tyres can no longer exert a lateral force, so the car becomes uncontrollable (Reif, 2014). It also causes significantly higher tyre degradation than if the wheels continue to turn during deceleration. Finally, locking up increases the stopping distance of the vehicle, as locking-up depends on the dynamic friction coefficient, which is lower than the static friction coefficient achieved when the wheel velocity is the same as the chassis velocity. Figure 1 shows that on a damp road with friction coefficient $\mu = 0.6$, the vehicle will lock up, and potentially cause an accident. The vehicle has a stopping distance of 143.5, which is defined as the area under the chassis velocity-time graph from 2 seconds onwards.

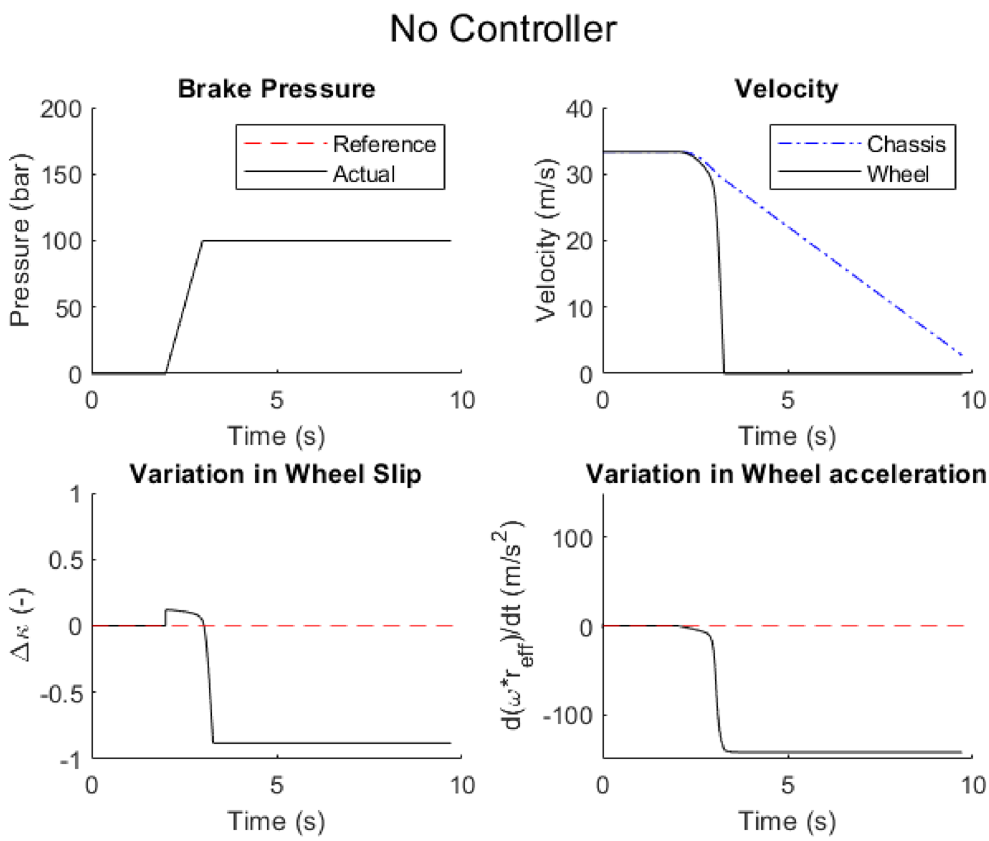

Figure 1: Performance without a controller

2. Wheel Slip Controller

The wheel slip controller aims to minimise the difference between the actual and reference wheel slip, with reference wheel slip value taken as 0.12.

$$\Delta\kappa = \kappa_{ref} - \kappa \tag{2}$$

A PID controller was implemented to control the braking of the vehicle. It received the wheel slip variation as an input and gave the braking pressure as an output.

$$p_{br} = K_p(V_x)\Delta\kappa + K_I(V_x)\int \Delta\kappa dt + K_d(V_x)\frac{d(\Delta\kappa)}{dt} \tag{3}$$

2.1 Optimisation

The controller was optimised by tuning the gains to minimise the stopping distance of the vehicle. Firstly, the optimum proportional gain was found. For this system, that was found to be between around 130, giving a stopping distance of 130.5m (Appendix 7.1), however the wheels still lock up. The optimum integral gain (with a proportional gain of 130) was around 160 (Appendix 7.1). This significantly reduced the stopping distance to 116.6m.

2.2 Gain Scheduling

The controller was further optimised by defining the gains as a function of the longitudinal velocity (V_x). As the velocity of the vehicle decreases, the fluctuations in $\Delta\kappa$ grow. Therefore, the proportional gain needs to decrease accordingly. Because of the non-linearity of the tyre, a power law relation was chosen, with B in the range of 0 to 2.

$$K_p(V_x) = AV_x^B, \quad K_I(V_x) = CV_x^D \tag{4}$$

To reduce the number of parameters from 4 to 2, the power law constant (A) was defined in terms of the initial velocity, the previously determined K_p and K_I and a power.

$$A = \frac{K_p}{V_{max}^B}, \quad C = \frac{K_I}{V_{max}^D} \tag{5}$$

The optimum parameters were [A,B,C,D] = [16,0.6,0.3,1.8], giving a stopping distance of 114.8m. Then the parameters A and C were further tuned to achieve a stopping distance of 112.2m (Table 1).

Adding a derivative gain had a small improvement on the performance of the vehicle, so a simple linear relation was chosen to minimise the number of parameters, and therefore reduce the time spent optimising, giving a stopping distance of 111.9m.

The system could have been further optimised by using multiple gain scheduling functions for different chassis velocities or using functions with more parameters (i.e. a polynomial function). However at this fidelity of optimisation, tuning the parameters was achieving diminishing returns. For example, adding a P controller reduced stopping distance by 13m, but the gain scheduled derivative only reduced it by 0.3m.

Table 1: Wheel Slip Controller Performance

K_p	$20V_x^{0.6}$
K_I	$0.7V_x^{1.8}$
K_d	$0.14V_x$
Stopping Distance	111.9m
Standard Deviation $\Delta\kappa$	0.0996

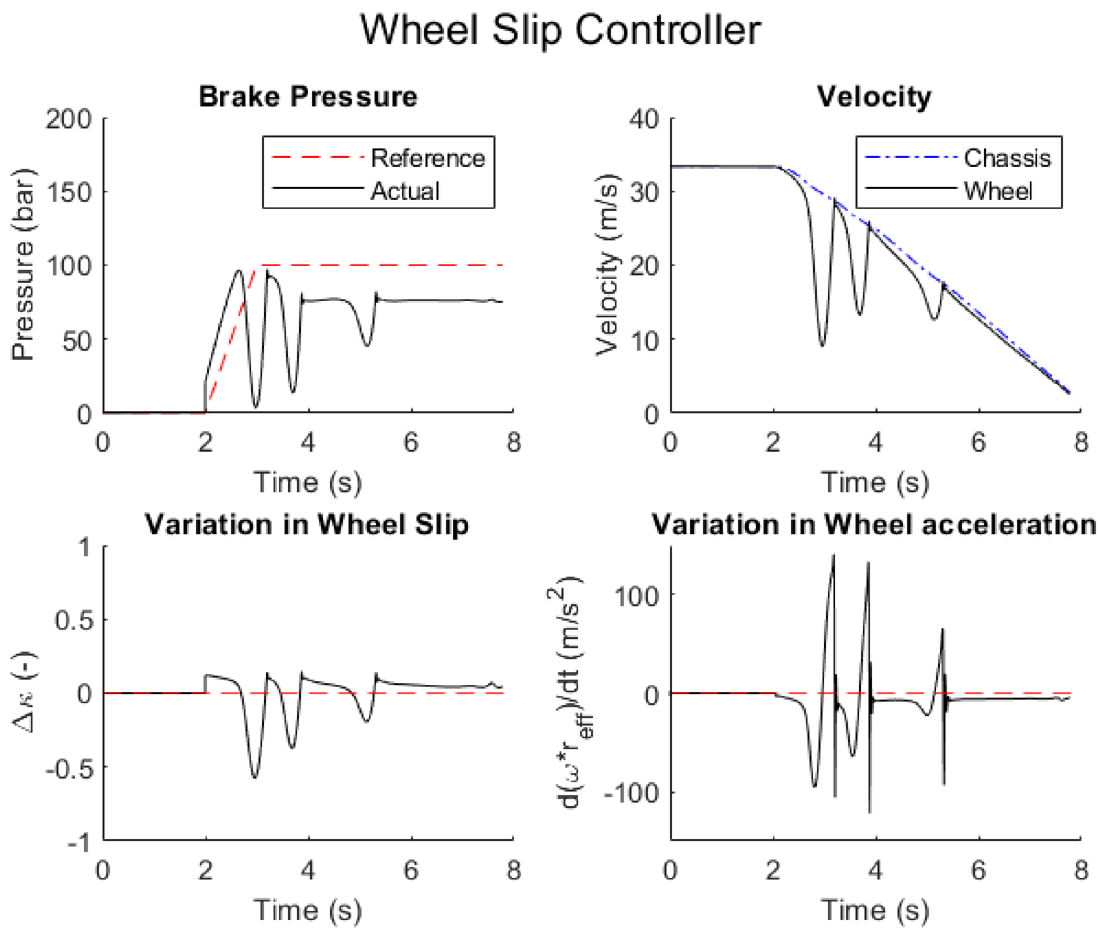

Figure 2: Optimised Wheel Slip Controller

3. Mixed Slip Controller

For the mixed slip controller, a P controller was used for the wheel slip, and a PI controller received the wheel acceleration as an input.

$$p_{br} = K_{p,1}(V_x)\Delta\kappa + K_{p,2}(V_x)\dot{\omega}r + K_I(V_x)\int \dot{\omega}rdt \quad (6)$$

With the additional wheel acceleration input, the performance of the mixed-slip controller could easily match that of the wheel slip-controller without much optimisation. Therefore, a linear gain scheduling function for proportional was sufficient, as was a constant integral gain.

Table 2: Mixed Slip Controller Performance

$K_{p,1}$	$9V_x$
$K_{p,2}$	$0.02V_x$
K_I	1.3
Stopping Distance	111.2m
Standard Deviation $\Delta\kappa$	0.0914

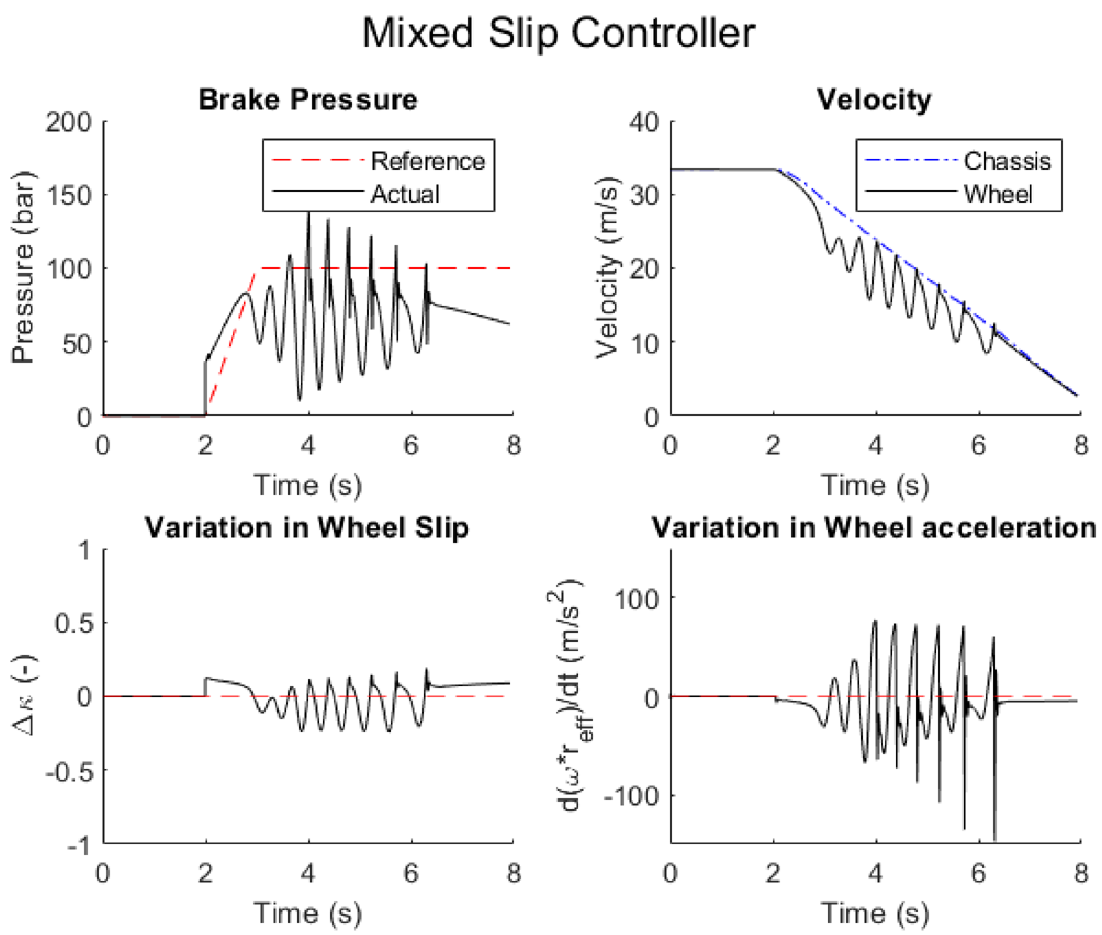

Figure 3: Mixed Slip Controller

4. Noise

Noise was added to the system, with $\kappa_n = \pm 0.025$ and $\dot{\omega}_n = \pm 0.5$ rad/s. The noise caused the tyres to lock-up at very low speeds for the wheel slip controller, whereas the noise had negligible effect on the mixed slip controller.

This is probably because the wheel acceleration noise was smaller than the wheel slip noise, relative to the respective output signal. Since the mixed slip is less effected by wheel slip input, it is less effected by the noise.

The stopping distance for the wheel slip controller increased by nearly a metre to 112.8m, whereas the stopping distance for the mixed slip controller remained at 111.2m.

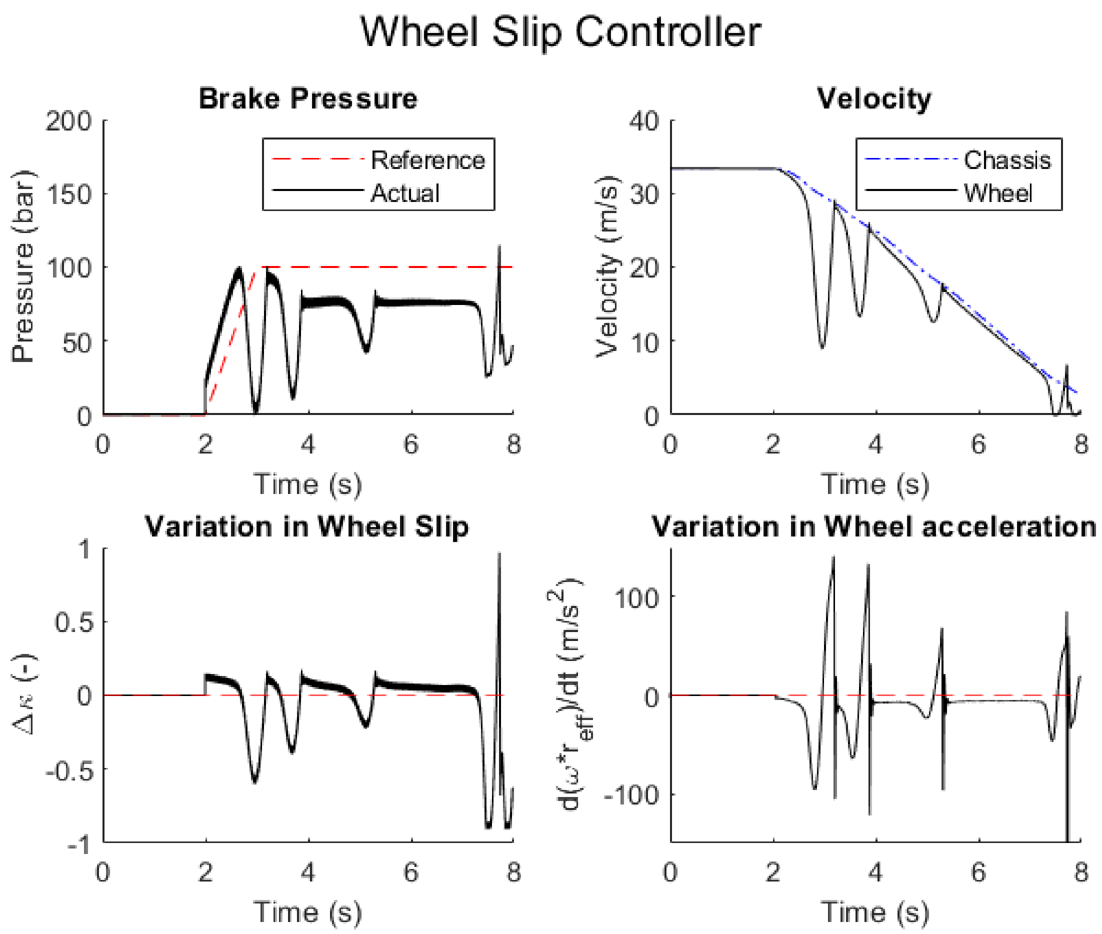

Figure 4: Wheel Slip Controller with Noise

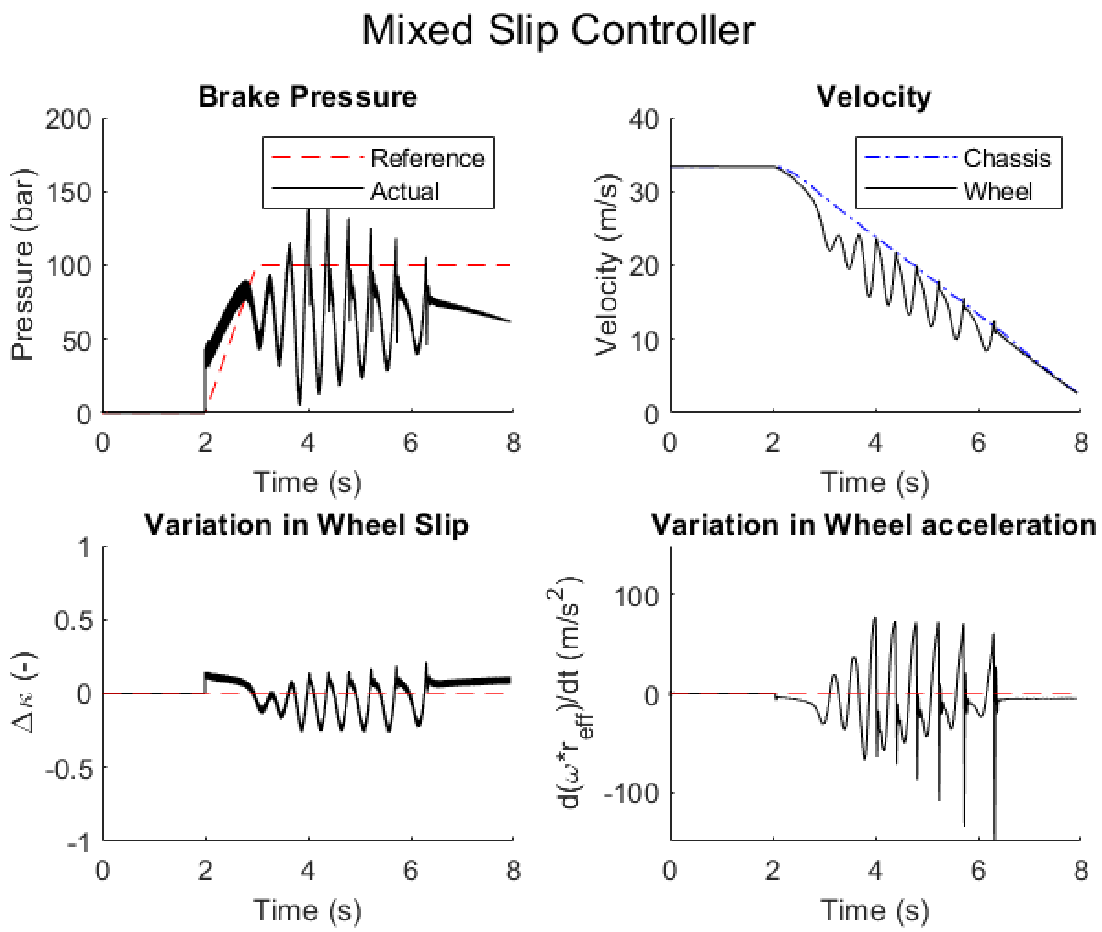

Figure 5: Mixed Slip Controller with Noise

5. Conclusion & Reflection

Overall, the mixed slip controller was superior to the wheel slip controller both with and without noise. Both controllers exhibit fluctuations in the applied brake pressure. Oscillations are beneficial in an emergency braking manoeuvre, because it is an obvious indication to the driver that the ABS is working. The mixed slip controller could also be further improved, by using a gain scheduled PID controller for both wheel slip and wheel deceleration. For example:

Table 3: Optimised Mixed Slip Controller Performance

$K_{p,1}$	$23V_x^{0.56}$
$K_{I,1}$	$0.7V_x^{2.25}$
$K_{d,1}$	$0.14V_x^{1.05}$
$K_{p,2}$	$0.021V_x^{1.06}$
$K_{I,2}$	$0.9V_x^{0.39}$
$K_{d,2}$	$0.0005V_x^{0.86}$
Stopping Distance	101.9m
Standard Deviation $\Delta\kappa$	0.054

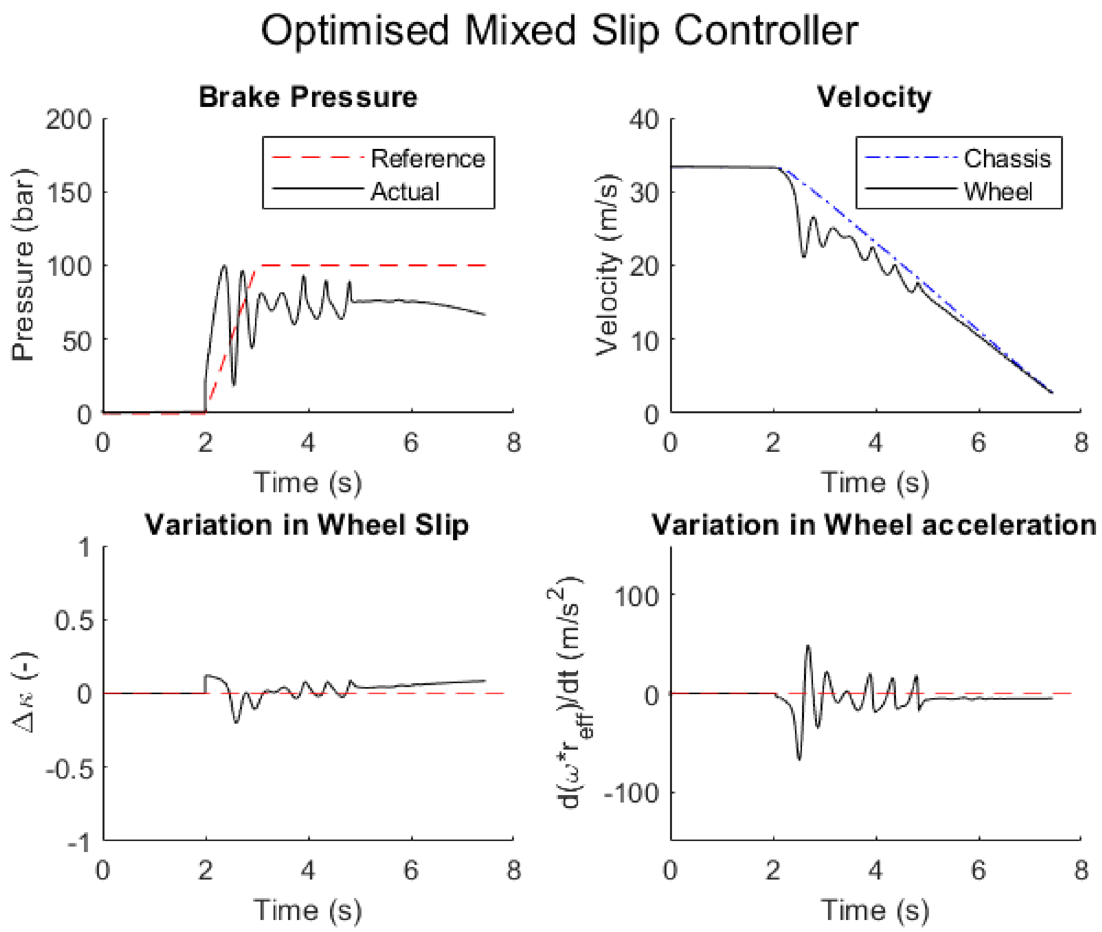

Figure 6: Optimised Mixed Slip Controller

5.1 Different Conditions

The mixed slip controller is superior for a friction coefficient of 0.6, however this does not mean that it is more effective than the wheel slip controller at different friction coefficients.

The ABS should operate effectively in a range of friction coefficients. The minimum coefficient of friction that causes the wheels to lock up is 0.79, so ABS is not required above this value. This corresponds to the lower limit for dry tarmac, which has a friction coefficient of 0.8 to 1.1 (Shyrokau, 2023).

The minimum friction coefficient that the controller is expected to perform effectively at is 0.4. This is the lowest friction coefficient for wet tarmac which is 0.4 to 0.7 (Shyrokau, 2023). Icy conditions were not considered because travelling at 120 km/h on ice or snow would be extremely reckless from the driver.

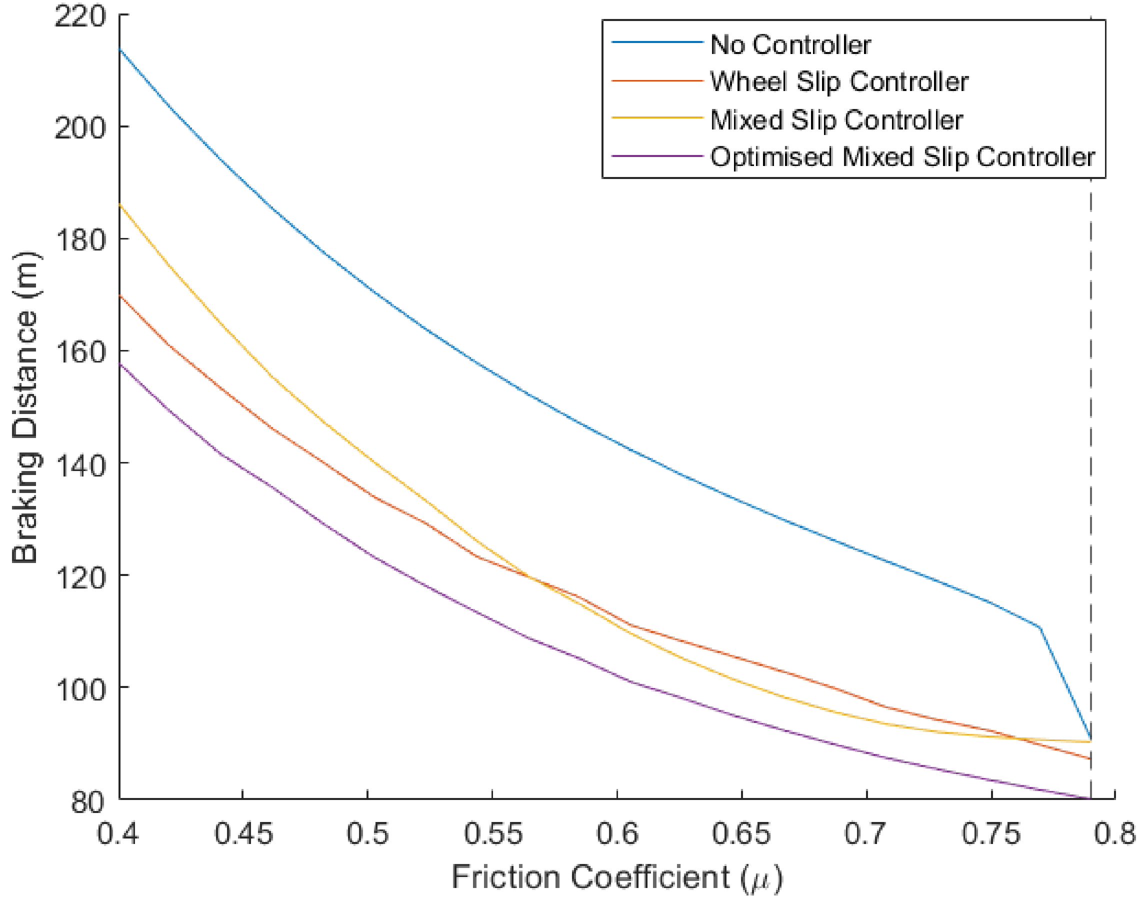

Figure 7: Stopping Distance as a function of friction coefficient

The mixed slip controller actually performs worse than the wheel slip controller at lower frictional coefficients, as it locks up for $\mu < 0.55$. This is not ideal, as the controller should be effective for all road surfaces so it could be argued the wheel slip controller should be selected as it is more robust. Additionally, the mixed slip controller requires a braking force exceeding 100 bar, which is not possible unless a motor is used to increase the braking pressure.

An improvement would be to use different controllers for different driving conditions (i.e. snow/gravel mode), but there would be insufficient time to switch modes during an emergency braking manoeuvre.

A better solution would be to use the optimised mixed slip controller, as the mixed slip controller has artificially not been fully optimised to be comparable to the wheel slip controller.

For all the controllers the applied pressure is greater than the reference pressure during the ramp phase. This is suitable for emergency braking where the minimum braking distance is the most important parameter, but in a nonemergency braking manoeuvre, intense braking will be uncomfortable for the passengers. Saturation could be applied to the controller, so the applied brake pressure can only be the same or lower than the reference pressure (Appendix 7.2).

5.2 Reflection

Overall, the most time-consuming part of this assignment was tuning the controller. Due to the relatively long time to simulate the vehicle's motion ($\approx$ 4 seconds on my computer), and the large number of controller parameters, optimisation by brute force was very slow. Therefore, each parameter was optimised independently or as a pair (in the case of the power law relation) to reduce computation time. However, each parameter is highly dependent on each other, so optimising all parameters simultaneously would probably lead to a more optimal controller. Also, a lack of understanding of the exact plant dynamics forced me to guess the gain scheduling function as a power law, whereas other functions may have been more suitable.

I also realised that the results I was getting may not be feasible as the applied brake pressure exceeded the reference pressure. However, as I had already done the time-consuming optimisation, it would take too long to redo it with brake pressure saturation. (Note: I have commented through the saturated block in Simulink and shown the effect of saturation in [Appendix 7.2]).

Overall, one of the main skills I developed during this assignment was using Simulink effectively, especially being able to control Simulink from a MATLAB script. Also, I developed my ability to efficiently tune a PID controller with gain scheduling, although I could further improve the controller by implementing a more advanced algorithm to find the optimal controller gains.

6. Table of Results

Table 4: Stopping Distance Results

	Without Noise (m)	**With Noise (m)**
No Controller	143.5	143.5
Wheel Slip Controller	111.9	112.8
Mixed Slip Controller	111.2	111.2
Optimised Mixed Slip Controller	101.9	102.1

7. Appendix

7.1 Wheel Slip Controller

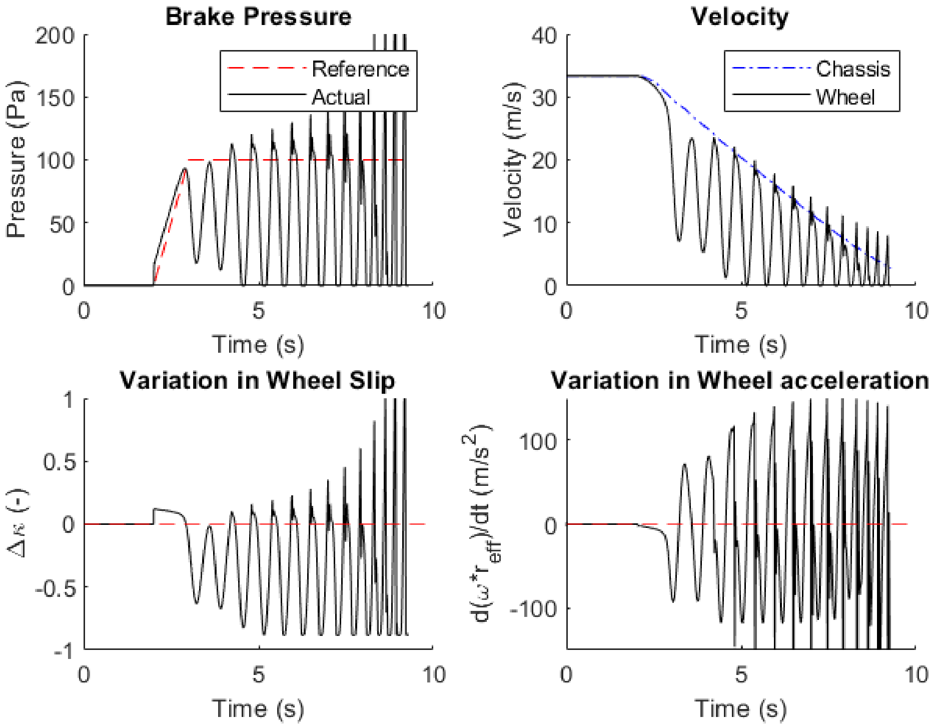

Figure 8: $K_p = 130,\ K_I = 0$

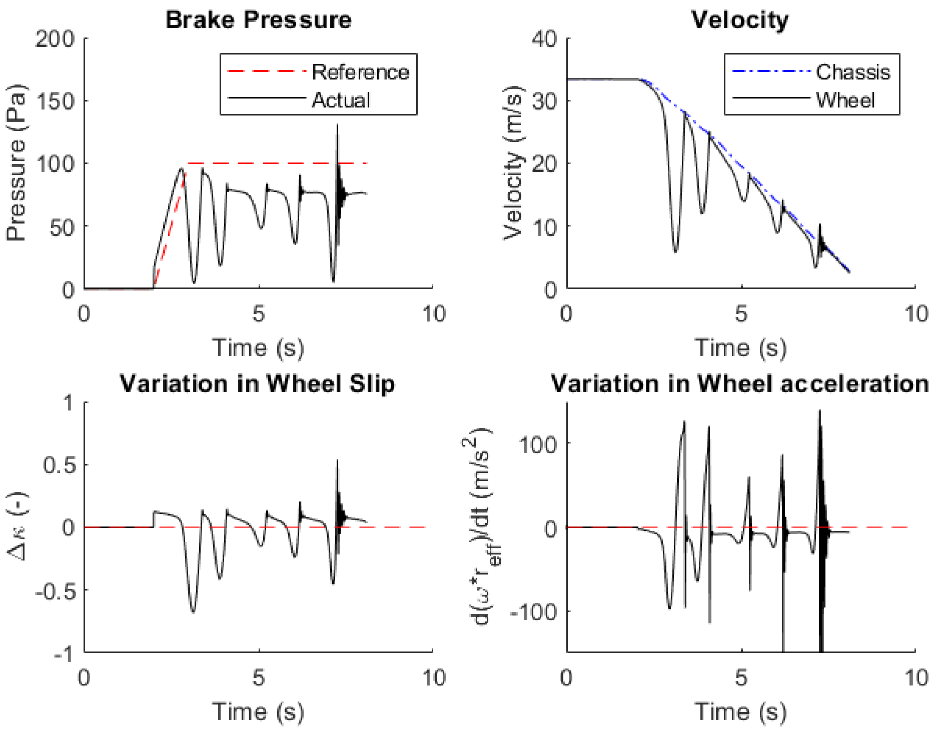

Figure 9: $K_p = 130,\ K_I = 160$

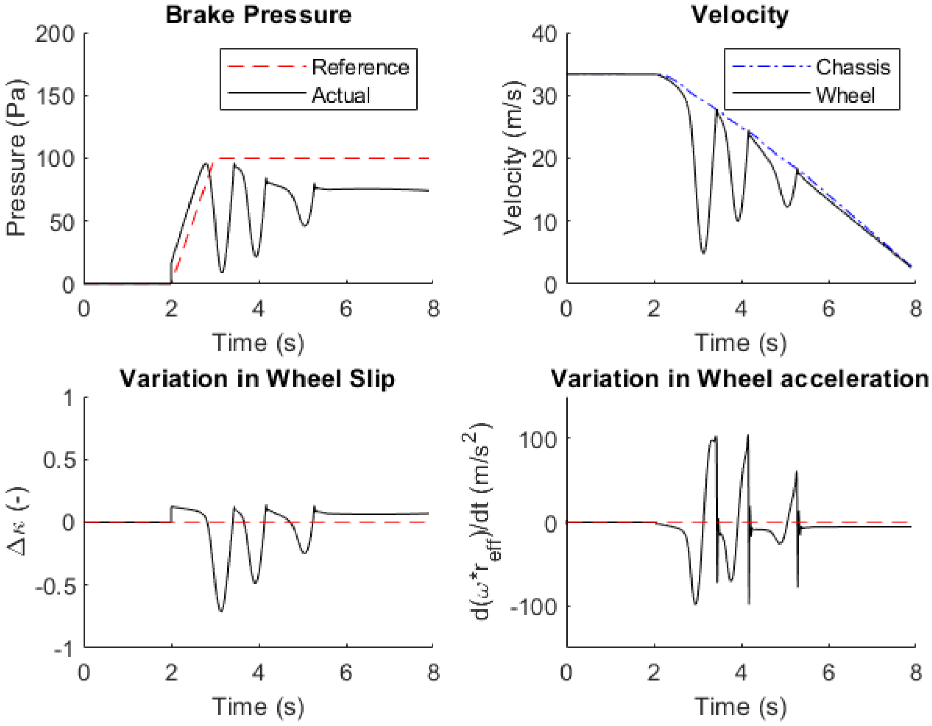

Figure 10: $K_p = 16V_x^{0.6}$, $K_I = 0.3V_x^{1.8}$

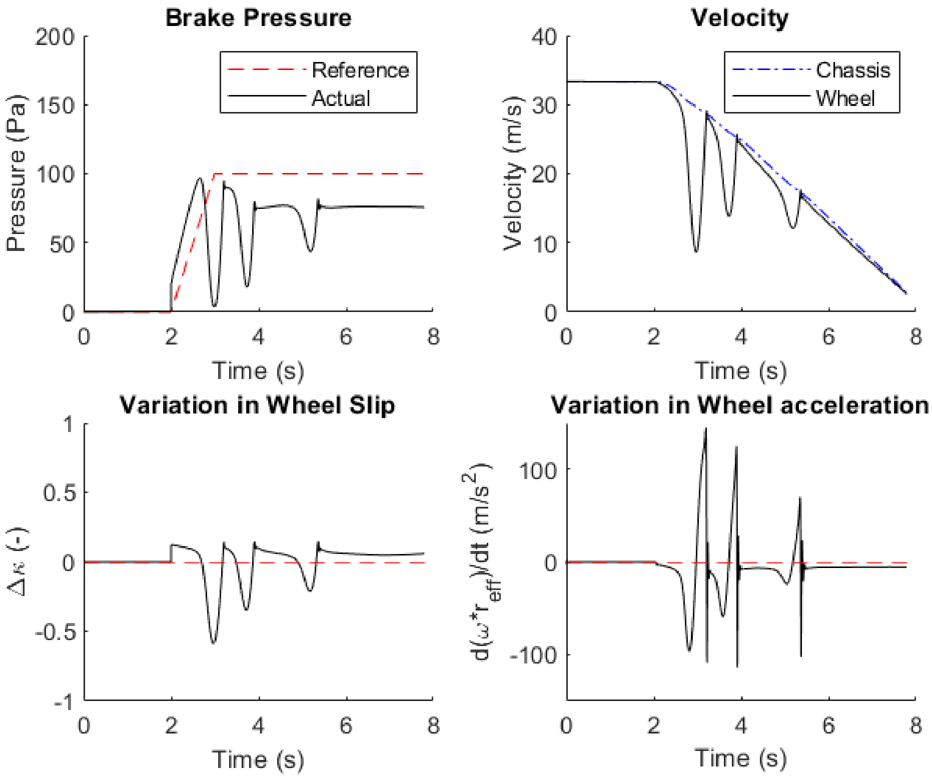

Figure 11: $K_p = 20V_x^{0.6}$, $K_I = 0.7V_x^{1.8}$

7.2 Mixed Slip Controller

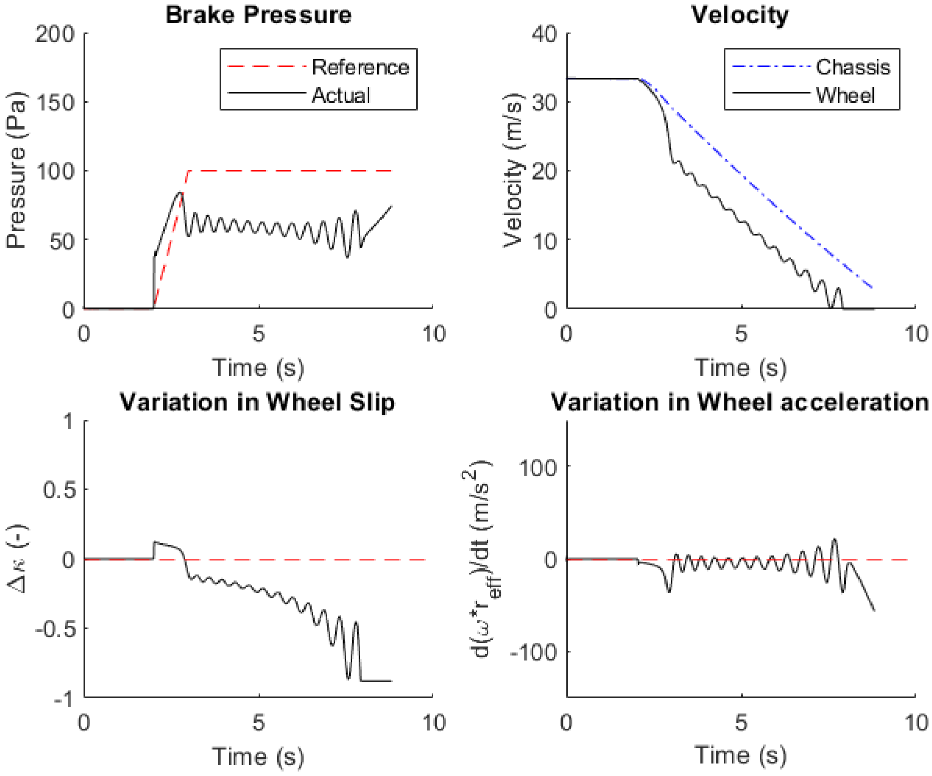

Figure 12: $K_{p,1} = 9V_x, K_{p,2} = 0.02V_x,\ K_I = 0$

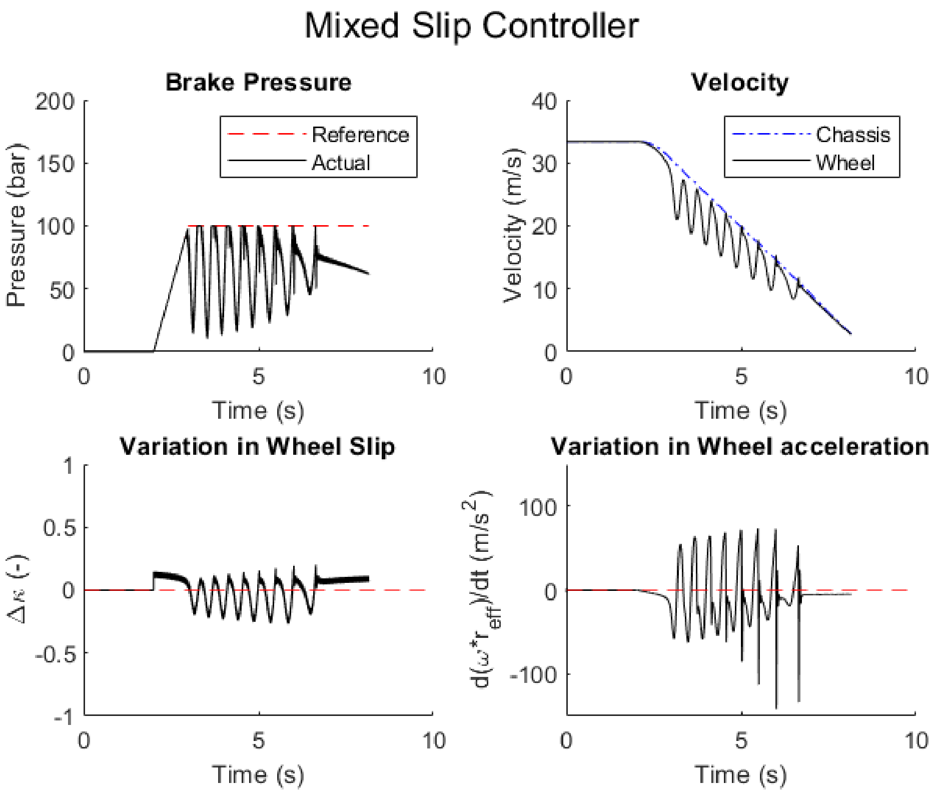

Figure 13: Mixed Slip Controller with noise and saturation

7.3 Controller Design

The wheel slip controller is in green, the mixed slip controller is in purple and the shared Simulink blocks are in blue (see below).

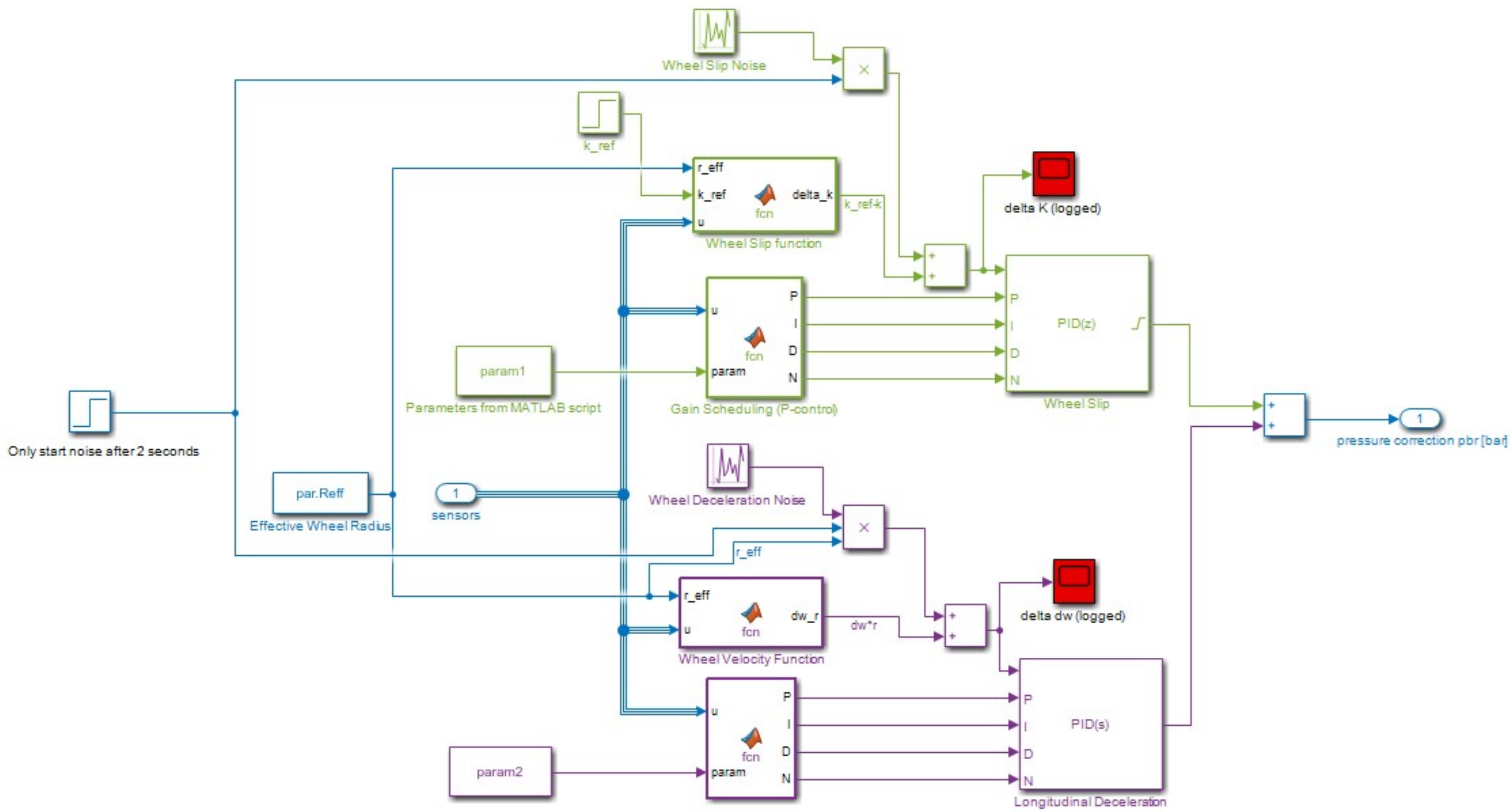

Figure 14: Controller Design

References

Reif, K. (2014). Brakes, brake control and driver assistance systems. *Weisbaden, Germany, Springer Vieweg*.

Shyrokau, B. (2023). *Tire dynamics & modelling: Part 1*.